***FINALLY*, WE GET TO DO SOME *REAL* TRIGONOMETRY !!!** 	**3 quizes R & D given starting Tue (#12). I graded 3rd quiz(on Wed). Then a "take home" quiz rewritting entire 6 fctns +=0 times; ck=2 times, 0 = 4 times, - = 5 times; = = 6 times Due Friday.**	EXAM CHAPTER 1 9/18	**#1** "Circle Secrets" Section I **Return exams** **Explain "narratives"** **Explain how to mark circles** 9/19	**#2** "Circle Secrets" Section II **Heston Lecture** 9/20
# 3 Circle Secrets III Pg 31 17, 18, 23, 24 note: sine is >0 & cosine < 0 in only 2nd quad **# 4** Pg 26 1 - 7, for 1 - 6 use cos add/sub formula then on pblm 7 ignore "properties" 9/ 23	**#5** "Circle Secrets" Section IV, V **Lecture #3** **DEF cot sec csc** **Range & Domain** 9/24	**#6** "Circle Secrets" Section VI Pg 55 18 – 23 Radical in Denom will be ok. No graphing. **Lecture #4** **tan add** 9/25	**# 7** Pg 71 1 – 13 **Lecture: "We don't need pg 66."** **Proof vs Verify** 9/26	**# 8** Pg 71 14 – 18 **#9** Pg 40 7 – 12 **We *always* prove!** 9/27
# 10 Pg 40 13 – 16 **# 11** Pg 67 1 – 11 **Have pblm 14 pg 71 put on board.** 9/30	**#12** "Circle Secrets" Section VII Quiz? **R & D of Fctns... "There are 6 trig fctns, this is one of them ..."** **Class corrects** **10 min** 10/1	**#13** Pg 71 - 72 22,23, 30, 31, 32, 34 **Q#2 R & D -- class corrects** 10/2	**#14** R/C Pg 44 **#15** Pg 67 14, 15, 17 **Put #12, 13 on boards** **Q#3 R & D -- I correct** Friday's Quiz: Prove a, b: a) $\sin\left(\frac{3\pi}{2} - x\right) = -\cos x$ b) Sec x Csc x = Tan x + Cot x c) Evaluate $\sin\left(\frac{-\pi}{12}\right)$ 10/3	**# 16** "Circle Secrets" Section VIII Quiz continued ... d) Find tan x given: $\sin(x) = \frac{3}{5},\ \frac{\pi}{2} < x < \pi$ **(15 min)** 10/4
# 17 Pg 45 2 - 6, 8 Pg 73 1, 2, 4, 5 10/ 7	Put Last Year's Exam on the Boards. 10/8	EXAM **CHAPTER 2** 2002 10/ 9	***We Love Trig !***	

I CIRCLE SECRETS

Honors Trig Chapter 2
Mr. Mumaugh

A) For each of the following, draw a circle indicating the given. State the coordinates and radians:

1) 30° 2) 120° 3) 210° 4) 330° 5) 135° 6) 45° 7) 180°
8) 315° 9) 60° 10) 210° 11) 225° 12) 90° 13) 150° 14) 0°

B) For each of the following, draw a circle indicating the given. State the coordinates and degrees:

1) $\frac{\pi}{3}$ 2) $\frac{3\pi}{4}$ 3) $\frac{5\pi}{3}$ 4) 2π 5) $\frac{\pi}{6}$ 6) $\frac{\pi}{2}$ 7) $\frac{7\pi}{4}$

8) $\frac{\pi}{4}$ 9) $\frac{2\pi}{3}$ 10) $\frac{5\pi}{4}$ 11) $\frac{3\pi}{2}$ 12) $\frac{11\pi}{6}$ 13) $\frac{5\pi}{6}$ 14) 0

C) For each of the following, draw a circle indicating the given. State the radians and degrees:

1) $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ 2) $(0,1)$ 3) $\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ 4) $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ 5) $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

6) $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ 7) $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ 8) $(-1,0)$ 9) $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ 10) $(0,1)$

11) $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ 12) $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ 13) $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ 14) $(1,0)$ 15) $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

II Write the general cosine add/subtract formula *EACH* time it is used!

A) Simplify: 1) $\cos\frac{5\pi}{12}$ 2) $\cos\frac{11\pi}{12}$ 3) $\cos\frac{13\pi}{12}$ 4) $\cos\frac{\pi}{12}$

5) $\cos\frac{-\pi}{12}$ 6) $\cos\frac{-13\pi}{12}$ 7) $\cos\frac{-11\pi}{12}$ 8) $\cos\frac{-5\pi}{12}$

B) WithOUT evaluating EACH factor in the original, evaluate: (hint: use Cos Add/Subtract)

1) $\cos\pi\cos\frac{2\pi}{3} + \sin\pi\sin\frac{2\pi}{3}$ 2) $\cos\frac{\pi}{6}\cos\frac{2\pi}{3} - \sin\frac{\pi}{6}\sin\frac{2\pi}{3}$

C) *USE the Cosine Add/Subtract formula* to **simplify as much as possible.**

1) $\cos(x+\pi)$ 2) $\cos(x-\pi)$ 3) $\cos\left(x+\frac{\pi}{2}\right)$ 4) $\cos\left(x-\frac{\pi}{2}\right)$ 5) $\cos(x+x)$

6) $\cos\left(\frac{x}{2}-\frac{x}{2}\right)$ 7) $\cos(0-x)$ 8) $\cos\left(\frac{x}{2}+\frac{x}{2}\right)$ 9) $\cos\left(\frac{\pi}{2}-x\right)$

III Write the general sine add/subtract formula *EACH* time it is used!

Simplify: 1) $\sin\frac{5\pi}{12}$ 2) $\sin\frac{11\pi}{12}$ 3) $\sin\frac{13\pi}{12}$ 4) $\sin\frac{\pi}{12}$

5) $\sin\frac{-\pi}{12}$ 6) $\sin\frac{-13\pi}{12}$ 7) $\sin\frac{-11\pi}{12}$ 8) $\sin\frac{-5\pi}{12}$

IV Draw the circle and properly mark the indicated radian measurement. Specifically state the sine of the measurement and the cosine of the measurement.

a) $\frac{5\pi}{6}$ b) $\frac{5\pi}{3}$ c) $\frac{-5\pi}{4}$ d) $\frac{-8\pi}{3}$ e) $\frac{13\pi}{12}$ f) $\frac{11\pi}{3}$ g) $\frac{-4\pi}{3}$ h) $\frac{-5\pi}{3}$

i) $\frac{11\pi}{2}$ j) $\frac{40\pi}{3}$ k) $\frac{-2\pi}{3}$ l) $\frac{-7\pi}{4}$ m) $\frac{11\pi}{6}$ n) $\frac{35\pi}{6}$ o) $\frac{-17\pi}{3}$ p) $\frac{-47\pi}{4}$

V Copy each of the tables. Fill in all the missing data. Show all work. No decimals, of course.

A)

x	$\sin x$	$\cos x$	$\tan x$	x	$\sin x$	$\cos x$	$\tan x$
0				π			
$\frac{\pi}{6}$				$\frac{7\pi}{6}$			
$\frac{\pi}{4}$				$\frac{5\pi}{4}$			
$\frac{\pi}{3}$				$\frac{4\pi}{3}$			
$\frac{\pi}{2}$				$\frac{3\pi}{2}$			
$\frac{2\pi}{3}$				$\frac{5\pi}{3}$			
$\frac{3\pi}{4}$				$\frac{7\pi}{4}$			
$\frac{5\pi}{6}$				$\frac{11\pi}{6}$			

B)

x	$\csc x$	$\sec x$	$\cot x$	x	$\csc x$	$\sec x$	$\cot x$
0				π			
$\frac{\pi}{6}$				$\frac{7\pi}{6}$			
$\frac{\pi}{4}$				$\frac{5\pi}{4}$			
$\frac{\pi}{3}$				$\frac{4\pi}{3}$			
$\frac{\pi}{2}$				$\frac{3\pi}{2}$			
$\frac{2\pi}{3}$				$\frac{5\pi}{3}$			
$\frac{3\pi}{4}$				$\frac{7\pi}{4}$			
$\frac{5\pi}{6}$				$\frac{11\pi}{6}$			

VI Copy the table. Fill in all the missing data. Show all work. No decimals, of course.

x	$\tan x$	x	$\tan x$	x	$\tan x$	x	$\tan x$
$\frac{\pi}{12}$		$\frac{9\pi}{12}$		$\frac{17\pi}{12}$		$\frac{25\pi}{12}$	
$\frac{2\pi}{12}$		$\frac{10\pi}{12}$		$\frac{18\pi}{12}$		$\frac{26\pi}{12}$	
$\frac{3\pi}{12}$		$\frac{11\pi}{12}$		$\frac{19\pi}{12}$		$\frac{27\pi}{12}$	
$\frac{4\pi}{12}$		$\frac{12\pi}{12}$		$\frac{20\pi}{12}$		$\frac{28\pi}{12}$	
$\frac{5\pi}{12}$		$\frac{13\pi}{12}$		$\frac{21\pi}{12}$		$\frac{29\pi}{12}$	
$\frac{5\pi}{12}$		$\frac{14\pi}{12}$		$\frac{22\pi}{12}$		$\frac{30\pi}{12}$	
$\frac{7\pi}{12}$		$\frac{15\pi}{12}$		$\frac{23\pi}{12}$		$\frac{31\pi}{12}$	
$\frac{8\pi}{12}$		$\frac{16\pi}{12}$		$\frac{24\pi}{12}$		$\frac{32\pi}{12}$	

VII Prove: a) $\sin^2\left(\frac{x}{2}\right) = \frac{1}{2}(1 - \cos x)$ b) $\left(\frac{1 + \tan x}{1 - \tan x}\right)^2 = \frac{1 + \sin 2x}{1 - \sin 2x}$

c) $\sin 3x = 3\sin x - 4\sin^3 x$ d) $\frac{1 - \tan^2 x}{1 + \tan^2 x} = 1 - 2\sin^2 x$

VIII Which (if any) of the following are identities?

a) $\tan^2 x - \sin^2 x = (\tan^2 x)(\sin^2 x)$ b) $\frac{\cos 4x}{1 + \sin 4x} = \sec 4x - \tan 4x$

c) $\frac{1}{6}(\sin 4x)(\cos 2x) - \frac{1}{3}\cos 4x \sin 2x = \frac{1}{3}\sin^3 2x$ d) $\sin x = \frac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}$

write onboard

Still tell your clients - start taking notes - watch and listen only for now.

$$\cos(x_2 - x_1) \stackrel{?}{=} \cos(x_2) - \cos(x_1)$$

Lecture #1

try $x_2 = x_1 = \frac{\pi}{2}$

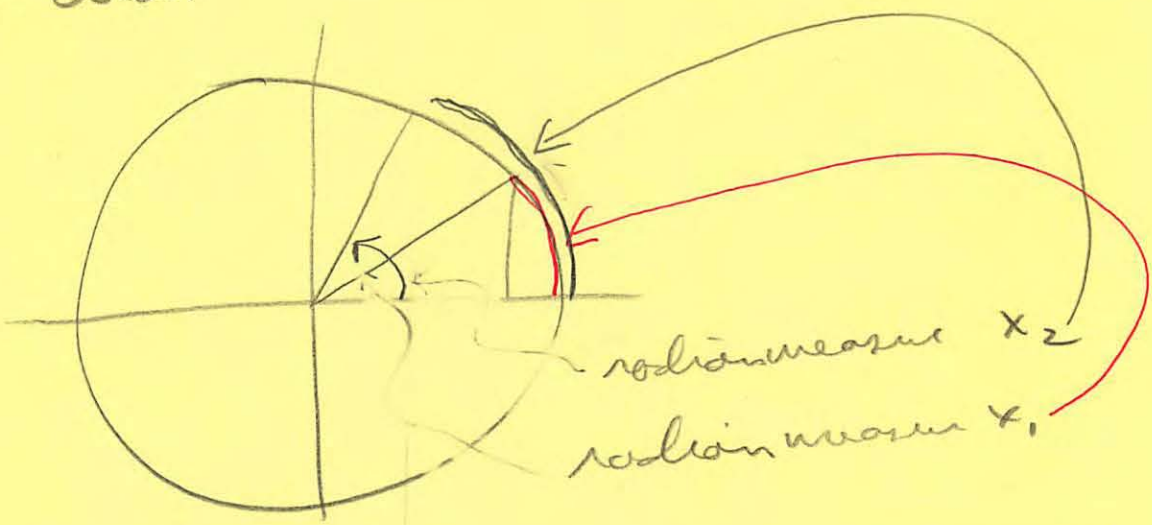
$$\cos\left(\frac{\pi}{2} - \frac{\pi}{2}\right) \stackrel{?}{=} \cos\left(\frac{\pi}{2}\right) - \cos\left(\frac{\pi}{2}\right)$$

$$\cos(0) = 0 - 0$$

$$1 = 0 \quad \text{NO}$$

!!

Draw circle



Now "digress" a:

How many of you saw the movie The Fugitive? I thought it was the Best movie of the Summer of 95. Did you see the scene in the subway.

In the ST. Patrick's day Parade You see Harrison sign on stage, then you see a Digital time/temperature clock. it reads 1:03 the next time you see it it reads 1:01 But it is in upper left corner of Picture and hard to see. finally you see it reads 12:47!

OPEN: DID You See "The Natural" on Television Last night?

Well, You have seen it sometim?

DID You see the glove Robert Redford had? a real pancake.

Speaking of Pancakes....

How about "Field of Dreams" Kevin Costner they had the wool uniforms and the pancake gloves.

Do you remember the scene where shoeless Joe Jackson is batting? usually it Show right handed batter. But Shoeless Joe Jackson was a Lefty!

other errors in films

Robin hood. wanted Posters nailed up on everything like the old west. the Entire Press had not been invented yet (200 yrs)
so they Never had wanted Poster.

Star wars scene of Battlestar and The X on the Bottom of spaceship. can only be seen using frame by frame.

Apollo 13 one engineer checks an astronaut's additions using a slide rule. Slide rules are Not used for addition.

XI $\sin^2 \star + \cos^2 \star = 1$

Ben Hur the "extras" were USC students. they were paid a Timex watch This was how Cecil B DeMille could have a "Cast of Thousands" Look at crowd. They are wearing their new watches! Yes.

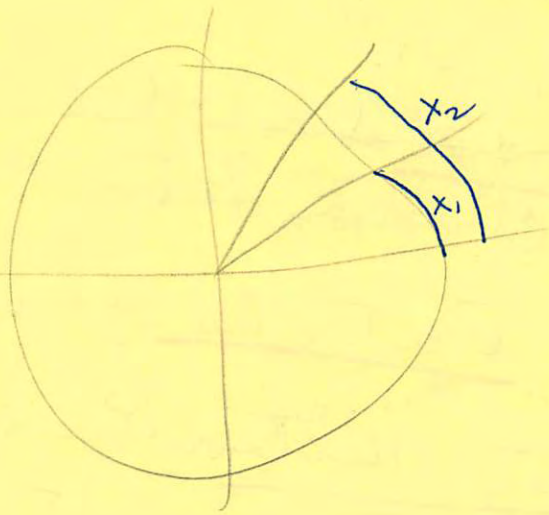
~~more people did during that film than any other.~~ Charleton Heston Drove a chariot chariots .... Tin tracks of film track.

another charleton Heston film was 10 commandment. they show it every year the Sunday before Easter.

charleton Heston is Moses. coming down from the mountain with the 10 commandments He sees the people worshiping a false idol Golden calf and throws the two stone tablets in disgust. well .... as He was walking down the mountain he was holding the tablets with this on the front I, II, ... X on the back ....

XI $\sin^2\theta + \cos^2\theta = 1$

the pairs were put back together ~~that~~ II was on the back so it got less notice than it should.



Now add arc lengths to picture

Open with "Heston Identity" story
Ben Hur Twin Cities, 10 commandments $s^2+c^2=1$
Tremendo Shoeless Joe Jackson was left handed but Batts right handed in field of Dreams

2-4

Addition Property of Cos. Robin Hood wanted Doctor Dirt pass 2004 US Letter

Question: is $\cos(x_2-x_1)=\cos x_2-\cos x_1$ for all values of x_2, x_1? NO;

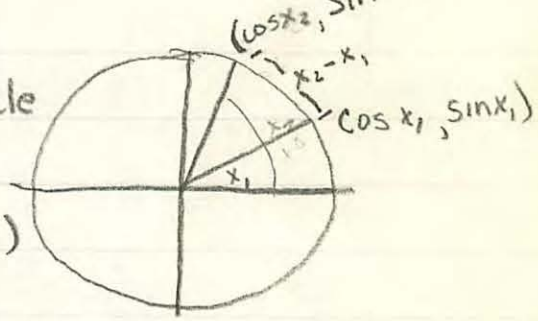
NOTE: tell students NOT to copy proof. WATCH the proof. I want to convince them of its validity not just them our method.

ie.: let $x_2=x_1=\frac{\pi}{2}$

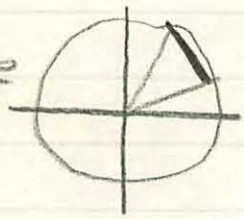
then $\cos(\frac{\pi}{2}-\frac{\pi}{2})=\cos\frac{\pi}{2}-\cos\frac{\pi}{2}$

$\cos(0)=0-0 \quad 1 \stackrel{?}{=} 0$

There is, however, a method for finding the relationship $\cos(x_2-x_1)=?$



First we have our unit circle
we have 3 arc lengths
ie. arc, has beginning coord. (1,0) and ending coord. $(\cos x, \sin x)$
and arc_2 begins (1,0) and ends $(\cos x_2, \sin x_2)$
arc_3 begins $(\cos x_1, \sin x_1)$ and ends $(\cos x_2, \sin x_2)$
Now draw the chord from $(\cos x_2, \sin x_2)$ to $(\cos x_1, \sin x_1)$
we will call that distance d_1.



The distance we are interested in is actually the arc distance. We will return to that at the end. Now we concentrate on the chord distance. What is this distance in terms of Sin & Cos?

Using the distance formula $d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

We Have:

$$d_1 = \sqrt{(\cos x_2 - \cos x_1)^2 + (\sin x_2 - \sin x_1)^2}$$

$$d_1 = \sqrt{\cos^2 x_2 - 2\cos x_2 \cos x_1 + \cos^2 x_1 + \sin^2 x_2 - 2\sin x_2 \sin x_1 + \sin^2 x_1}$$

regrouping we have:

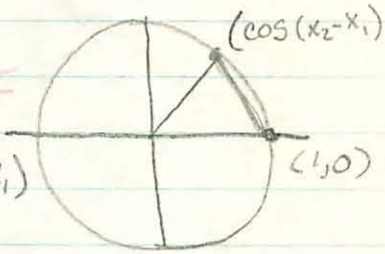
$$d_1 = \sqrt{\cos^2 x_2 + \sin^2 x_2 + \cos^2 x_1 + \sin^2 x_1 - 2\cos x_2 \cos x_1 - 2\sin x_2 \sin x_1}$$

using the ~~Heat~~ ~~ing~~ identity $C^2 + S^2 = 1$; we have:

$$d_1 = \sqrt{1+1-2\cos x_2 \cos x_1 - 2\sin x_2 \sin x_1}$$

$$d_1 = \sqrt{2 - 2\cos x_2 \cos x_1 - 2\sin x_2 \sin x_1}$$

Now draw another unit circle mark out the same arc_3 but begin at $(1,0)$ and end at $\cos(x_2-x_1), \sin(x_2-x_1)$



Again draw the chord. Notice that this arc length is constructed to be the same as arc_3. The chord here has a distance we will call d_2

$$d_2 = \sqrt{(1-\cos(x_2-x_1))^2 + (0-\sin(x_2-x_1))^2}$$

$$d_2 = \sqrt{1 - 2\cos(x_2-x_1) + \cos^2(x_2-x_1) + \sin^2(x_2-x_1)}$$

$$d_2 = \sqrt{1+1-2\cos(x_2-x_1)}$$

$$d_2 = \sqrt{2-2\cos(x_2-x_1)}$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

Eg $\cos\left(\frac{7\pi}{12}\right) = ?$

consider $\frac{3}{12} + \frac{4}{12} = \frac{7}{12}$

So

$$\cos\left(\frac{7\pi}{12}\right) = \cos\left(\frac{3\pi}{12} + \frac{4\pi}{12}\right)$$

$$= \cos\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$$

$$= \cos\frac{\pi}{4}\cos\frac{\pi}{3} - \sin\frac{\pi}{4}\sin\frac{\pi}{3}$$

no test

$$= \frac{\sqrt{2}}{2}\,\frac{1}{2} - \frac{\sqrt{2}}{2}\,\frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$$

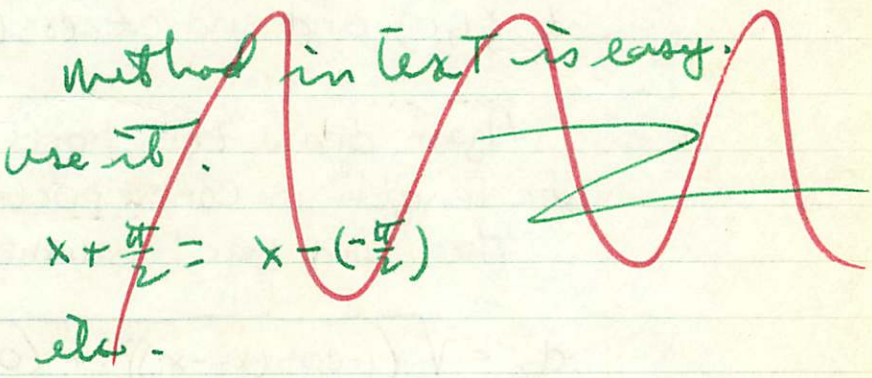
$$= \frac{\sqrt{2} - \sqrt{6}}{4}$$

End of Lecture #1

method in text is easy. use it.

$x + \frac{\pi}{2} = x - (-\frac{\pi}{2})$

etc.



Now Notes Begins

From Geometry you know that equal arcs have equal chords we know d_1 must $= d_2$ Thus.

$$\sqrt{2-2\cos x_2 \cos x_1 - 2\sin x_2 \sin x_1} = \sqrt{2-2\cos(x_2-x_1)}$$

Square both sides to get;

$$2-2\cos x_2\cos x_1 - 2\sin x_2 \sin x_1 = 2-2\cos(x_2-x_1)$$

$$-2\cos x_2\cos x_1 - 2\sin x_2 \sin x_1 = -2\cos(x_2-x_1)$$

Multiply by $\frac{1}{2}$ $\cos(x_2-x_1) = \cos x_2 \cos x_1 + \sin x_2 \sin x_1$

General Cos ± form $\cos(A_2 \pm B_1) = \cos A_2 \cos B \mp \sin A \sin B$

Now that we have spent so much time on the cos subtraction formula what good is it?

~~See example Pg 58 #1~~

~~Demo~~

Next replace $x_2 = 0$ and $x_1 = x$ to get $\cos(-x) = \cos x$ This is clear when one looks at the unit circle as well. Remember $-x$ means measure x radians clockwise.

replace $x_2 = \frac{\pi}{2}$ and $x_1 = x$ to get $\cos(\frac{\pi}{2}-x) = \sin x$

replace $x_2 = \frac{\pi}{2}$ and $x_1 = x$ to get $\sin(\frac{\pi}{2}-x) = \cos x$

In like manner (as in text) we get the identity $\sin(-x) = -\sin x$ Definition of "odd fctn"

and Cos addition formula,

$$\cos(x_2+x_1) = \cos x_2 \cos x_1 - \sin x_2 \sin x_1$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B.$$

Students are to write this as notes

~~H.W. Pg 61-63 1-8 ALL~~

~~17-26 ALL~~

Now do demo on Preceding Page.

$$\cos\left(x \pm \tfrac{\pi}{2}\right) = \cos x \cos\tfrac{\pi}{2} \mp \sin x \sin\tfrac{\pi}{2}$$
$$= \mp \sin\tfrac{\pi}{2}$$

$$\cos\left(0 \pm (x \pm \tfrac{\pi}{2})\right) = \cos 0 \cos\left(x \pm \tfrac{\pi}{2}\right) \mp \sin 0 \sin(\quad)$$
$$0$$

$$\cos\left(\tfrac{\pi}{2} - x\right) = \cos\tfrac{\pi}{2}\cos x + \sin\tfrac{\pi}{2}\sin x$$
$$0 \quad + \sin x$$

Let $X = A - B$

$$\cos\left(\tfrac{\pi}{2} - (A-B)\right) = \sin(A-B)$$

$$\cos\left(\left(\tfrac{\pi}{2} - A\right) + B\right) =$$

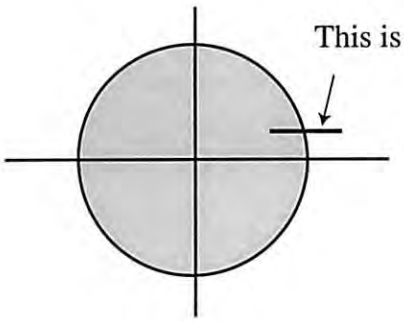
$$\cos\left(\tfrac{\pi}{2} - A\right)\cos B - \sin\left(\tfrac{\pi}{2} - A\right)\sin B =$$

$$\sin A \cos B - \sin B \cos A =$$

we want

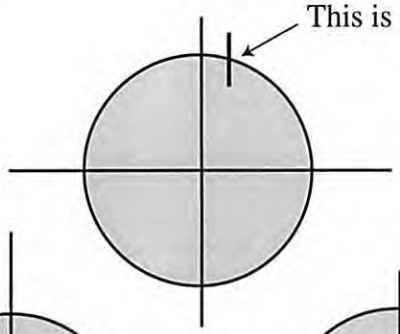
If we knew this we wouldn't need $\sin A$,

Circles ***must*** be marked in this fashion:



This is $\frac{\pi}{6}$ which is 30°.

All $\frac{\star}{6}$ fractions are marked with a horizontal line.

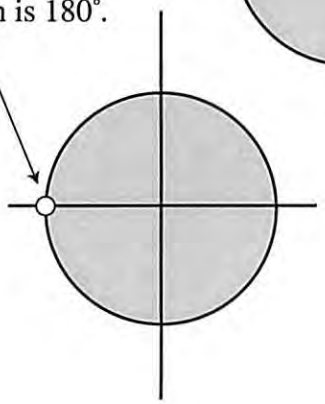


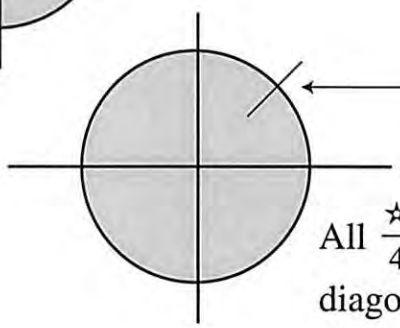
This is $\frac{\pi}{3}$ which is 60°.

All $\frac{\star}{3}$ fractions are marked with a vertical line.

This is π which is 180°.

Axis intercepts are marked with circles.



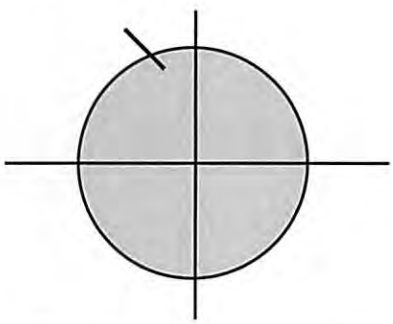


This is $\frac{\pi}{4}$ which is 45°.

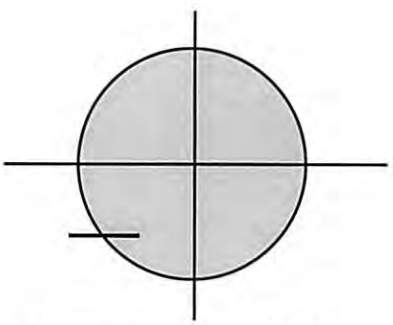
All $\frac{\star}{4}$ fractions are marked with a diagonal line.

Quiz.....

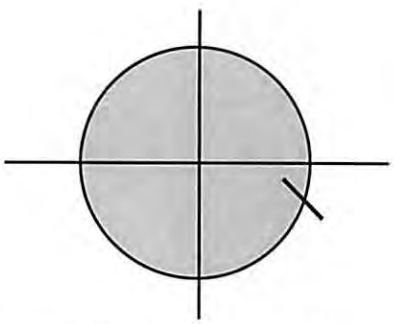
What radian measures are indicated?



This is poorly marked because it is marked too high. It is intended to represent $\frac{3\pi}{4}$ because the mark is neither vertical nor horizontal.



This is poorly marked as well because it is marked too low. It is intended to represent $\frac{7\pi}{6}$ because the line marking the circle is horizontal.



Again a poorly marked circle this time because it is marked too high. This one is intended to represent $\frac{7\pi}{4}$ because the line is neither horizontal nor vertical.

The *Heston Identity*

Mumaugh

The most basic of all trigonometry identities is the Heston Identity: $\sin^2 \bigstar + \cos^2 \bigstar = 1$

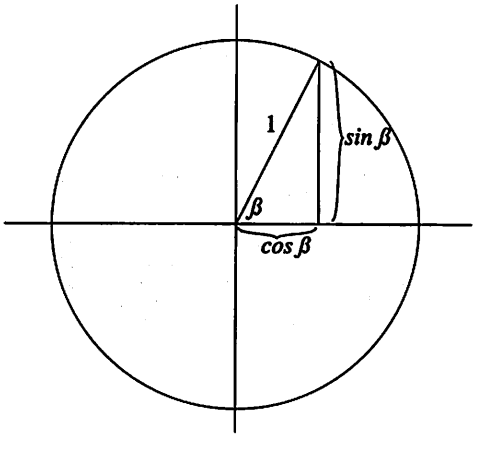
Translated to English, the Heston Identity says if you square the sine of some arc length and add that to the square of the cosine of the *same* arc length, the result will be exactly 1. You can replace the "arc length" with "degree measure" and the result is exactly the same.

One of the first questions which arise when confronted with the *Heston Identity* is, "Why is it called the *Heston Identity* ? The answer is quite simple. In this day and age it is quite common for computer programs to contain things called "Easter eggs". "Easter eggs" are usually cute or otherwise unusual pictures which are included in computer programs. To "activate" or see an Easter egg, you usually must option-click on particular place in a programs "about" window. Sometimes you will see a list of contributors to the program, sometimes a picture of something significant to the programmers. Macintosh programs seem to have an unusual number of these Easter eggs. Back in the 1960's the idea was to include extra, non related stuff in motion pictures. Sometimes this occurred by accident and sometimes it was planned. Now, motion picture studios approach companies and ask them if they would like their product displayed in a new movie and of course there would be a "consideration fee" to be paid to the motion picture studio. In the movie E.T. ---- What was dropped so that E.T. could follow the trail? Reese's Pieces. Do you think it was an accident they used Reese's Pieces rather than, say M & M's ?

I digress.... Anyway...

In the movie 10 Commandments staring Charleton Heston, on the back of one of the tablets he brought down to the people, you can clearly see $\sin^2 \bigstar + \cos^2 \bigstar = 1$ carved. Some people believe that Cecil B. DeMille was honoring his math teacher wife with that and others say that one of the prop people (who was a student at U.S.C.) put it in as a prank ordered by his fraternity house but in any case, it has since been known as the *Heston Identity.*

The second question usually asked has to do with why does *Heston's Identity* works. Basically, it works because of the Pythagorean theorem -- that is the "sum of the squares of the sides a right triangle = the square of the hypotenuse". In our case, our hypotenuse will be 1 --- the radius of the "unit circle".



Then next logical question has to do with what we use we make of the *Heston Identity.* All that and more on the next page.....

One of the big uses of the *Heston Identity* comes when we prove two very useful formulas. The actual proofs are in your text. Since we are doing this "lecture" by long distance, we will not go into the particulars. We shall use a little demonstration to show the usage and value of the *Heston Identity* and two of the direct results of the *Heston Identity*.

Question: Is cosine distributive? that is... Does: $\cos(x_2 - x_1) = \cos x_2 - \cos x_1$?

We consider $x_2 = \frac{\pi}{2}$ and $x_1 = \frac{\pi}{2}$ so we get:

$$\cos\left(\frac{\pi}{2} - \frac{\pi}{2}\right) = \cos\frac{\pi}{2} - \cos\frac{\pi}{2}$$

$$\cos(0) = \cos\frac{\pi}{2} - \cos\frac{\pi}{2}$$

$$1 = 0 - 0$$

Clearly $1 \neq 0$ so cosine (and no other trig function for that matter) is distributive. That leads us to seek out what $\cos(x_2 - x_1) = \cos x_2 - \cos x_1$ is equal to. ....

To make a long story shorter ....

We have two important formulas

The "Cosine Add/Subtract" : $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

I read it as.... cosine of A plus or minus B equals cosine of the first, cosine of the second *disagree* sine of the first sine of the second. The *disagree* refers to the sign. If we had $\cos\left(\frac{2\pi}{3} + \frac{\pi}{4}\right)$ we would use the formula like this:

$$\cos\left(\frac{2\pi}{3} + \frac{\pi}{4}\right) = \cos\frac{2\pi}{3}\cos\frac{\pi}{4} - \sin\frac{2\pi}{3}\sin\frac{\pi}{4}$$

$$= -\frac{1}{2}\cdot\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$$

$$\cos\left(\frac{11\pi}{12}\right) = \frac{-\sqrt{2} - \sqrt{6}}{4}$$

The second formula is "The Sine Add/Subtract" formula: $\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A$

$$\sin\left(\frac{2\pi}{3} + \frac{\pi}{4}\right) = \sin\frac{2\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\cos\frac{2\pi}{3}$$

$$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\cdot\left(\frac{-1}{2}\right)$$

$$\sin\left(\frac{11\pi}{12}\right) = \frac{\sqrt{6} - \sqrt{2}}{4}$$

This formula is read: sine of A plus or minus B is sine of the first, cosine of the second *agree* sine of the second, cosine of the first.

Now, try using your calculator.... $\left(\frac{-\sqrt{2} - \sqrt{6}}{4}\right)^2 + \left(\frac{\sqrt{6} - \sqrt{2}}{4}\right)^2 = ?$

From: "Scott E. Meyer" <se-meyer@northwestern.edu>
Subject: **Thanks!**
Date: March 30, 2004 9:22:47 AM PST
To: Jim <teeitup98@sbcglobal.net>

Jim,

Thanks for your answer. And you are certainly right that many, if not all, movies have some kind of weird errors.

As it turns out, so does The Ten Commandments. About 12 years ago, the DeMille family gave me a grant and permission to delve into the DeMille archives, since I was taking a close look at the research behind the film. One of the more interesting things I learned was that the tablets are missing the commandment about not taking the name of the Lord in vain! The stone had been brought from Egypt (near one of the mountain thought by some to be Mount Sinai), and was already engraved by the time the error was spotted.

Thanks again for clearing up this mystery.

Scott

At 09:12 AM 3/30/2004 -0800, you wrote:

> Hello Scott.
>
> You are correct. The "Heston Identity" was an inside joke that I used for about 25 years at Vacaville High School as the advanced mathematics teacher.
>
> In the trigonometry class on the day we started the add/subtract identities for sine and cosine I wanted to make the 20 minute lesson last 50 minutes and make it memorable for the students as well.
>
> The way I did that was to start the lecture on the cosine addition formula. After the picture was drawn and I began the pitch about this arc length is equal to that arc length, etc. I digressed -- almost in an "absent minded professor" mode -- and ask if the students had seen what ever the latest hot movie was. I eventually aimed the commentary to old films. I talked about Ben Hur and the chariot races and the tire tracks on the course next to the chariots. I talked about the extras wearing Timex watches. Another comment was about the original Star Wars movie. There was a one-frame error in that film and I talked about how to find it. The movie The Fugitive had some good errors to talk about as well. In fact, virtually every film made has some kind of quirky error so it is usually easy to pick a current film to start the story.
>
> Finally I would get to 10 Commandments. The way I told the story was that Charlton Heston walked down from the burning bush with the 10 Commandments and when he saw how the people were behaving, he threw the tablets to the ground and they shattered. I told the students, much like the Star Wars movie, if you single step, frame by frame, you will see on the back of one of the tablets 11. $\sin 2x + \cos 2x = 1$. In some years I told the students that one of the people on the set was honoring his mother, a math teacher, with by inserting this into the film. Other years I made up other stories about how it got there such as an "Easter Egg" like many Macintosh computer programs had in the 90s. What you found on my web site was some lecture notes that I prepared for my students when I was out of class on that critical day.
>
> Ever since that day, the world has referred to that formula as the Heston Identity.
>
> Actually referring to it as the Heston Identity made it easy to mention and the students knew exactly what I meant

when I did. For example, when they wanted help with a proof and they missed the pythagorean connection, I would say that "Heston would cry!".

Anyway, I hope that clears up the origin of the Heston Identity for you. I am surprised that it has spread so much. I guess that is an indicator of the efficiency of the webcrawlers the search engines use since I have never submitted those pages to any search engine.

Thanks for asking and best of luck to you.

Jim Mumaugh
(retired math teacher)

On Mar 30, 2004, at 7:31 AM, Scott E. Meyer wrote:

> To whom it may concern:
>
> at the URL:
>
> http://mumaugh.us/heston/trig_folder/Heston_Identity.Day%202.pdf
>
> I noticed a reference to something called "The Heston Identity." This phrase also appears elsewhere on the web. I assume this is meant to be a joke, since the equation, "sin2x+cos2x=1", is evidently considered a Pythagorean identity, and nothing is seen (clearly or otherwise) on the backs of either of the tablets in the film, The Ten Commandments.
>
> Or have I missed something?
>
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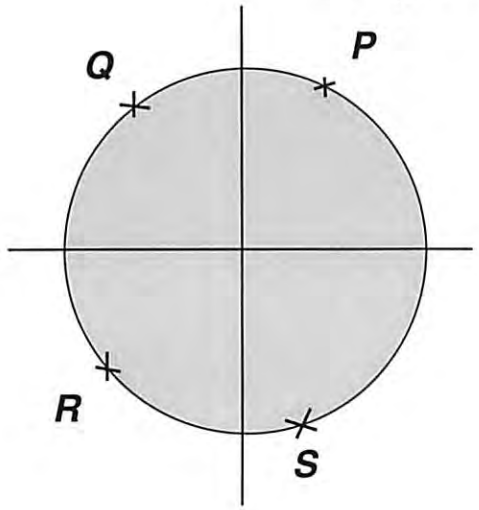
Day 3 notes



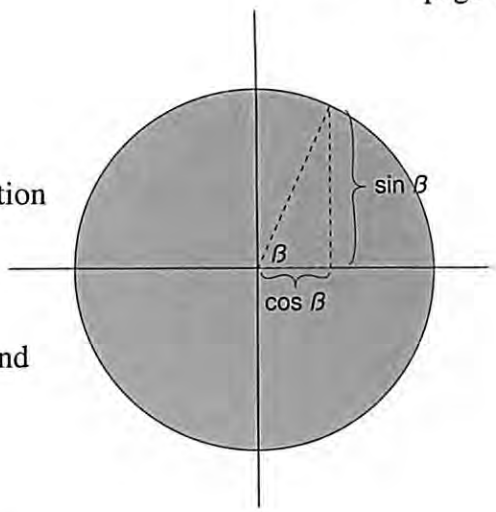
	If	Then
1)	$\cos A > 0$ $\sin A < 0$	P Q R S
2)	$\cos A < 0$ $\sin A < 0$	P Q R S
3)	$\cos A > 0$ $\sin A > 0$	P Q R S
4)	$\cos A < 0$ $\sin A > 0$	P Q R S

In other words....

Knowing only the sine of a value isn't enough to tell in which quadrant we are referencing. ....

You must *also* know the cosine of the value....

Day 4 Notes



$A(x) = \cos x$

D_x $x \in R$ because we can go "around the circle in either direction as much or as little as we like.
ie. No restrictions.

R_A $-1 \le \cos x \le 1$. The least cosine can be is -1 (when $x = \pi$) and the most cosine can be is $+1$ when $x = 0$.

Period 2π.

To prove any function has period "a", you *must* show that $f(x+a) = f(x)$.

example: $\cos(x+2\pi) = \cos x \cos 2\pi - \sin x \sin 2\pi$

$= \cos x\ (1) - \sin x\ (0)$

$\cos(x+2\pi) = \cos x$

Thus, cosine has period 2π. This is reasonable because 2π represents a full revolution around the unit circle so one revolution from point X puts us back at the same spot.
Cosine is an even function.
A function is called even if $f(-x) = f(x)$. An even function "absorbs the negative sign." If you looked at the graph of an even function, you would see that the graph on the right side of the origin would look like a reflection of the graph on the left side of the origin.

example: $\cos(0-x) = \cos 0 \cos x + \sin 0 \sin x$

$= (1)\cos x\ - (0)\sin x$

$\cos(-x) = \cos x$

so cosine is an even function.

$B(x) = \sin x$

D_B $x \in R$ for the same reason that cosine has a domain of all real numbers.

R_B $-1 \le \sin x \le 1$. See the picture listed with cosine. Sine is simply the vertical distance from the x-axis to the point in question.

Period 2π

odd function. A function is said to be an odd function if $f(-x) = -f(x)$. In an odd function, we can "factor out" the negative sign. The graph of an "odd function" reflects through the origin.

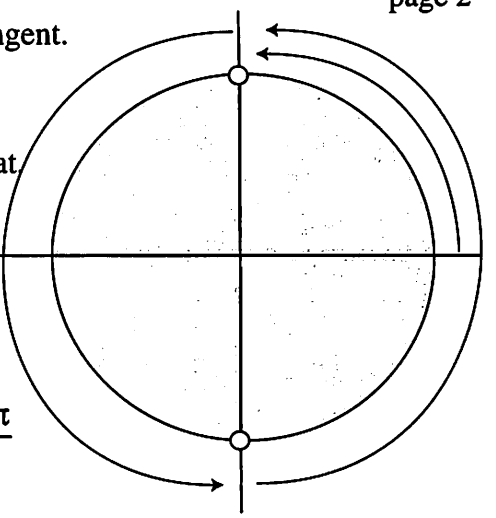
$\sin(0-x) = \sin 0 \cos x - \sin x \cos 0$

$= 0\cos x - \sin x\ (-1)$

$\sin(-x) = -\sin x$

so sine is an odd function.

$C(x) = \tan x$ which is defined to be the fraction: $\dfrac{\sin x}{\cos x}$ is known as Tangent.



Clearly, we cannot allow cosine to equal zero since division by zero is undefined. $\cos x = 0$ when $x = \dfrac{\pi}{2}$ and then every π length after that. We start at zero.... We go $\dfrac{\pi}{2}$ to the first spot then π to the next spot, then π to the following spot, etc. We abbreviate this as $\dfrac{\pi}{2} + k\pi$ or $\dfrac{(2k+1)\pi}{2}$

$D_C \quad x \notin \left\{\dfrac{(2k+1)\pi}{2}\right\}$ and $R_D \quad \tan x \ \varepsilon \ R$

consider the fraction $\dfrac{\sin x}{\cos x}$. Let's consider the first quadrant so we are looking at positive values. The numerator has a maximum of one and a minimum of zero. The denominator has a maximum of one. The important item is that the denominator gets *smaller* thus you will be dividing by a smaller number making the quotient larger. Of course the smallest value the fraction can be is zero (when the numerator is zero and the denominator is 1). So, from zero, the value of tangent gets larger and larger without bound. In the second quadrant, the tangent is negative because sine is positive but cosine is negative (making the fraction negative). The magnitude can get large but its value will be negative. The conclusion is that the range of tangent is that $R_C \quad \tan x \ \varepsilon \ R$.

The period of tangent is not 2π like it is for sine and cosine. Remember, to prove a function has period "a", you must show that $f(x+a) = f(x)$.

$$\begin{aligned}
\tan(x+\pi) &= \frac{\sin(x+\pi)}{\cos(x+\pi)} \\
&= \frac{\sin x\cos\pi + \sin\pi\cos x}{\cos x\cos\pi - \sin x\sin\pi} \\
&= \frac{\sin x(-1) + 0\cos x}{\cos x(-1) - \sin x(0)} \\
&= \frac{-\sin x + 0}{-\cos x - 0} \\
&= \frac{\sin x}{\cos x} \\
\tan(x+\pi) &= \tan x
\end{aligned}$$

The "co" functions

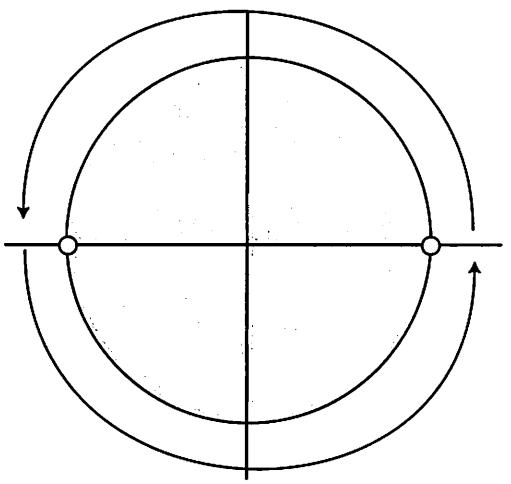
$D(x) = \sec x$ which is called secant and is equal to $\frac{1}{\cos x}$.

Notice this function has the same situation as tangent, that is, the cosine is on the bottom therefore the secant function has the same domain restrictions as tangent. $D_D \quad x \notin \left\{\frac{(2k+1)\pi}{2}\right\}$

The range of secant has an important difference from tangent. The numerator is one. A fraction is smallest when its denominator is at its largest and the largest cosine can be is 1 so $\frac{1}{1} = 1$ is the smallest (positive) value attainable. Of course, secant can be negative. In that case the largest negative value which can be attained
is – 1.. Thus, the range of secant is given by: $R_D \quad \sec x \le -1$ or $\sec x \ge +1$
The period is 2π as is cosine and also like cosine, secant is an even function.

$E(x) = \csc x = \frac{1}{\sin x}$ and is known as cosecant.

The domain restrictions are computed in a similar fashion but with different results than they were computed for tangent and secant. We start at the first zero which is $x = 0$ and then travel π distance to the next (and all other zeros of the function).



so $0 + k\pi$

$k\pi$

Thus, $D_E \quad x \notin \{k\pi\}$

$R_E \quad \csc x \le -1$ or $\csc x \ge +1$ by the same reasoning as for the secant function.

period is 2π

Odd function.

$F(x) = \cot x = \frac{\cos x}{\sin x}$ known as cotangent. $D_F \quad x \notin \{k\pi\}$ because sine is in the denominator like csc x.

$R_F \quad \cot x \in R$ for the same reasons as for tan x. Period is π and cotangent is an odd function.

$a(x)=\cos x$	D_a	$x \in R$	R_a	$-1 \le \cos x \le 1$	2π even
$b(x)=\sin x$	D_b	$x \in R$		$-1 \le \sin x \le 1$	2π odd
$c(x)=\tan x$	D_C	$x \notin \left\{\frac{(2k+1)\pi}{2}\right\}$	R_D	$\tan x \in R$	π odd
$d(x)=\sec x$	D_d	$x \notin \left\{\frac{(2k+1)\pi}{2}\right\}$	R_d	$\sec x \le -1$ or $\sec x \ge +1$	2π even
$e(x)=\csc x$	D_e	$x \notin \{k\pi\}$	R_e	$\csc x \le -1$ or $\csc x \ge +1$	2π odd
$f(x)=\cot x$	D_f	$x \notin \{k\pi\}$	R_D	$\cot x \in R$	π odd

An example of what is meant by "evaluate":

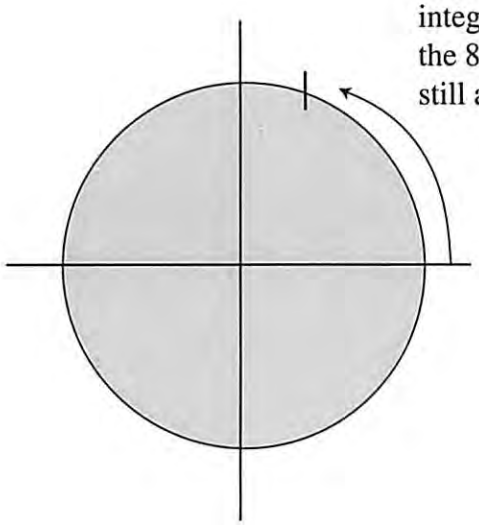
Evaluate $\tan\frac{25\pi}{3}$

$\tan 8\frac{1}{3}\pi$

$\tan\frac{\pi}{3}$

$\frac{\sin\frac{\pi}{3}}{\cos\frac{\pi}{3}}$

$\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$



Since tangent has period π, we can throw out any integer multiples of the angle. In this case, we toss out the 8π which is just 4 revolutions anyway and we are still at the same spot.

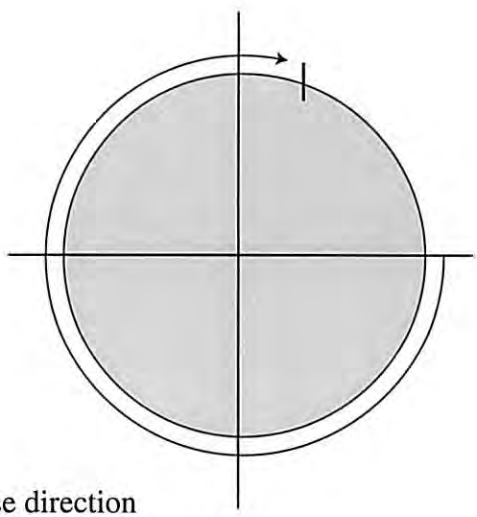
Evaluate $\sec\left(-\frac{5\pi}{3}\right)$

$= \sec\frac{\pi}{3}$

$= \frac{1}{\cos\frac{\pi}{3}}$

$= \frac{1}{\frac{1}{2}}$

$= 2$



Notice that we measured in a clockwise direction because the arc measurement was negative.

H Trig

Day 5 notes

Mr. Mumaugh

Open with 6 Trig fctns & D, R. odd/even then.

We have a sine add/subtract formula $\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A$
and we have the cosine add/subtract formula: $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

so you know there has to be a tangent add/subtract formula. The bad news is that there are secant, cosecant and cotangent add/subtract formulas too. The good news is that we don't bother with them but we do know and love the tangent add/subtract formula:

$$\tan(A \pm B) = \frac{\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A}{\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B}$$

$$= \frac{\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A}{\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B} \cdot \frac{\frac{1}{\cos A \cos B}}{\frac{1}{\cos A \cos B}}$$

$$= \frac{\frac{\sin A \cos B}{\cos A \cos B} \pm \frac{\sin B \cos A}{\cos A \cos B}}{\frac{\cos A \cos B}{\cos A \cos B} \mp \frac{\sin A \sin B}{\cos A \cos B}}$$

$$= \frac{\frac{\sin A}{\cos A} \pm \frac{\sin B}{\cos B}}{1 \mp \frac{\sin A \sin B}{\cos A \cos B}}$$

$$= \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

Define odd/even fctn

Is the tangent an odd, an even (or neither) function?

$$\tan(0 - x) = \frac{\tan 0 - \tan x}{1 - \tan 0 \tan x}$$

$$= \frac{0 - \tan x}{1}$$

$$= -\tan x$$

so tangent is an odd function.

Tangent has period π. We saw that yesterday.... but with the tangent add/subtract formula it is much cleaner...

$$\tan(x + \pi) = \frac{\tan x + \tan \pi}{1 - \tan x \tan \pi}$$

$$= \frac{\tan x + 0}{1 - \tan x(0)}$$

$$= \frac{\tan x}{1}$$

$$= \tan x$$

A couple of fun examples on the next page.....

Given: $\tan x = 3 \qquad \pi < x < \frac{3\pi}{2}$

Find $\sin 2x$

$$\frac{\sin x}{\cos x} = 3$$
$$\sin x = 3\cos x$$
$$\sin^2 x = 9\cos^2 x$$
$$\sin^2 x = 9(1-\sin^2 x)$$
$$\sin^2 x = 9 - 9\sin^2 x$$

because of the Heston Identity... $S^2 + C^2 = 1$ so $C^2 = 1 - S^2$.

$$10\sin^2 x = 9$$
$$\sin^2 x = \frac{9}{10}$$
$$\sin x = \pm\frac{3}{\sqrt{10}} = \pm\frac{3\sqrt{10}}{10}$$

but x is in the third quadrant so we want $-\frac{3\sqrt{10}}{10}$

furthermore, since the Heston Identity says

$S^2 + C^2 = 1$

so $\left(\frac{9}{10}\right) + \cos^2 x = 1$

$$\cos^2 x = \frac{1}{10}$$
$$\cos x = \pm\frac{1}{\sqrt{10}}$$

since we are in the 3rd quadrant, cosine is negative so we want

$$\cos x = -\frac{1}{\sqrt{10}}$$

finally we are ready to compute $\sin 2x$....

since $\sin 2x = \sin(x+x) = \sin x\cos x + \sin x\cos x = 2\sin x\cos x$

so.... $2 \cdot \frac{-\sqrt{3}}{\sqrt{10}} \cdot \frac{-1}{\sqrt{10}}$

$\frac{2\sqrt{3}}{10} = \frac{\sqrt{3}}{5}$ and the final answer is..... $\frac{\sqrt{3}}{5}$

convenient values ....

Let's find the x when we know that $\tan x = -\frac{\sqrt{3}}{3}$.

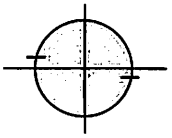
1) Let's consider the number $\frac{\sqrt{3}}{3}$. We know that $\frac{\sqrt{3}}{3}$ is the answer you'd get when you rationalized $\frac{1}{\sqrt{3}}$.

2) $\sqrt{3}$ ..... $\sqrt{3}$ ..... when you last saw a $\sqrt{3}$ in this class, what number was *under* it? 2? Yes that is $\frac{\sqrt{3}}{2}$.

3) $\frac{\sqrt{3}}{2}$ ..... $\frac{\sqrt{3}}{2}$ .....when you last saw a $\frac{\sqrt{3}}{2}$ in this class, what number was *with* it? $\frac{1}{2}$?

Yes that is, $\frac{\sqrt{3}}{2}$ and $\frac{1}{2}$ are "companions" aren't they?

4) Tangent is negative in the 2nd and 4th quadrants so that means x in this problem is in the 2nd or 4th quads.

5) so we have $-\frac{\sqrt{3}}{3}$ which is $-\frac{1}{\sqrt{3}}$ which is $-\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$ which is $\frac{\sin x}{\cos x}$ thus:  $x \in \left\{\frac{5\pi}{6}, \frac{11\pi}{6}\right\}$

BASIC IDENTITIES

For each $x \subset \mathcal{R}$ for which both members are defined:

A From the definitions of the circular functions:

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x} = \frac{1}{\tan x}$$

$$\sec x = \frac{1}{\cos x} \qquad \csc x = \frac{1}{\sin x}$$

B By referring to the identity $\cos^2 x + \sin^2 x = 1$, we can derive the two identities:

$$1 + \tan^2 x = \sec^2 x \qquad \cot^2 x + 1 = \csc^2 x$$

C $$\cos\left(\frac{\pi}{2} - x\right) = \sin x \qquad \sin\left(\frac{\pi}{2} - x\right) = \cos x$$

D
$$\cos(-x) = \cos x \qquad \sec(-x) = \sec x$$
$$\sin(-x) = -\sin x \qquad \csc(-x) = -\csc x$$
$$\tan(-x) = -\tan x \qquad \cot(-x) = -\cot x$$

E
$$\cos(x_2 \pm x_1) = \cos x_2 \cos x_1 \mp \sin x_2 \sin x_1$$
$$\sin(x_2 \pm x_1) = \sin x_2 \cos x_1 \pm \cos x_2 \sin x_1$$
$$\tan(x_2 \pm x_1) = \frac{\tan x_2 \pm \tan x_1}{1 \mp \tan x_2 \tan x_1}$$

F
$$\cos(x \pm \pi) = -\cos x \qquad \cos(\pi - x) = -\cos x$$
$$\sin(x \pm \pi) = -\sin x \qquad \sin(\pi - x) = \sin x$$
$$\tan(x \pm \pi) = \tan x \qquad \tan(\pi - x) = -\tan x$$

G From the identities in Group E, for $x_2 = x_1 = x$ and for $+$:

$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$
$$\sin 2x = 2 \sin x \cos x$$
$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

H $$\cos\frac{x}{2} = \pm\sqrt{\tfrac{1}{2}(1 + \cos x)} \qquad \sin\frac{x}{2} = \pm\sqrt{\tfrac{1}{2}(1 - \cos x)} \qquad \tan\frac{x}{2} =$$

We Don't Need No Stinkin' Pg 66

Open your text to page 66..... a wealth of information!. The first question that comes to mind is, "Why is this page *still* in the text book? .... Such an encyclopedia of information begs to be torn from the book and hung in a student's bedroom so he/she can see it upon awakening in the morning, just before drifting off to sleep at night and other inspirational times of the day.

But wait ....

Let's look closer .....

Section A ..... obviously!

Section B ..... $S^2 + C^2 = 1$ then divide by S^2 to get $1 + \cot^2 x = \csc^2 x$

or divide by $\cos^2 x$ to get $\tan^2 x + 1 = \sec^2 x$

We *already* know the Heston Identity so this section is obvious!

Section C ..... If I wanted to know what $\cos\left(\frac{\pi}{2} - x\right)$ was ....

I'd use the cosine add/subtract formula I already know! Obvious!

Section D ..... Obvious. Cosine and secant are even ... they "absorb the negative", the other trig functions are called odd functions.

Section E ..... This looks impressive but our version is more readable. We will use *our* version. Obviously!

Section F ..... See the comment for section C. Obvious.

Section G ..... Why memorize a zillion formulas? We know $2x = x + x$ and use the appropriate add identity. Obvious.

Section H ..... Useful ... on occasion. Think .... $\frac{x}{2} + \frac{x}{2} = x$ and you can generate these in a flash. Almost obvious.

So.... the conclusion is.....

We know $\sin^2 x + \cos^2 x = 1$

$\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos(A \pm B) = \cos A \cos B \pm \sin A \sin B$ and

$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$ and generally, that is all we really need to know.

What we really need is practice using these identities and we have plenty of that coming up right now...

The textbook offers two versions of "practice". **Verify** and **Prove** are the two types. The main difference between them is that when you **verify** an identity, you are allowed to manipulate both sides of the identity and in the case of a **Prove**, you are (generally) not allowed to alter one side or the other. To make it less confusing, we *never verify*. We **always prove**. There are two distinct versions of **Prove**.

Lecture #2

3rd day of Second Exam Lecture.

Sin add

$$\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A$$

this can be developed from cos ± formula.

Notice $\cos(0-x) = \cos x$ Yes, Today

Try $\sin(0-x) = ?$

$\sin(x+\pi)$

$\sin(x-\pi)$

$\sin(x+\frac{\pi}{2})$

$\sin(x+x)$

Demo pblm 18 pg 31

$$\sin(x_2 - x_1) = \sin x_2 \cos x_1 - \sin x_1 \cos x_2$$

$$= \frac{12}{13}\,\frac{4}{5} - \frac{3}{5}\left(\frac{-5}{13}\right)$$

$$= \frac{48+15}{65} = \frac{63}{65}$$

what quad? I or II

$$\cos(x_2 - x_1) = \cos x_2 \cos x_1 + \sin x_2 \sin x_1$$

$$\frac{4}{5}\left(\frac{-5}{13}\right) + \frac{12}{13}\,\frac{3}{5}$$

$$\frac{-20+36}{65} > 0 \therefore \text{I quad.}$$

Quiz Ch 2

4 + √3 0 − 2 = 0

1. $\sin\left(\frac{3\pi}{2} - x\right) = -\cos x$

$\sin\frac{3\pi}{2}\cos x + 0$

$-\cos x$ QED $x \in \mathbb{R}$

2. $\sin(x+y) + \sin(x-y) = 2\sin x \cos y$

$\sin x\cos y + \sin y\cos x + \sin x\cos y - \sin x\cos y =$

$2\sin x\cos y =$

QED $x \in \mathbb{R}$

3. $\sec x \csc x = \tan x + \cot x$

$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$

$\frac{\sin^2 x + \cos^2 x}{\sin x\cos x}$

$\frac{1}{\cos x \sin x}$ QED

$\sec x \csc x$ $x \neq \left\{\frac{k\pi}{2}\right\}$

4. $\sin\left(\frac{-\pi}{12}\right) - \sin\left(\frac{3\pi}{12} - \frac{4\pi}{12}\right) =$

$\sin\frac{\pi}{4}\cos\frac{\pi}{3} - \sin\frac{\pi}{3}\cos\frac{\pi}{4}$

$\frac{\sqrt{2}}{2}\frac{1}{2} - \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} = \frac{\sqrt{2} - \sqrt{6}}{4}$

5. $\tan x = ?$ $\sin x = \frac{3}{5}$ Quad II

$\cos x = -\frac{4}{5}$ $\therefore \tan = \frac{\frac{3}{5}}{\frac{4}{5}} = -\frac{3}{4}$

(1) $\sin \frac{5\pi}{6}$

$\sin(\pi - \frac{\pi}{6}) = \sin \frac{\pi}{6}$ $\frac{1}{2}$

(2) $\sin \frac{13\pi}{12}$ $\sin\left(\frac{9\pi}{12} + \frac{4\pi}{12}\right) =$

$\sin(\frac{\pi}{12} + \pi) = -\sin \frac{\pi}{12} = -\sin \frac{\pi}{3} - \frac{\pi}{4}$

$-\left[\sin \frac{\pi}{3} \cos \frac{\pi}{4} - \sin \frac{\pi}{4} \cos \frac{\pi}{3}\right]$

$-\left[\frac{\sqrt{3}}{2} \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \frac{1}{2}\right]$

$\frac{\sqrt{2} - \sqrt{6}}{4}$

(3) $\sin \frac{11\pi}{3}$

$\sin(\frac{-\pi}{3} + 4\pi)$

$\sin(\frac{-\pi}{3})$

$-\sin \frac{\pi}{3} = -\frac{\sqrt{3}}{2}$

(4) $\cos \frac{5\pi}{3}$

$\cos(-\frac{\pi}{3} + 2\pi)$

$\cos(-\frac{\pi}{3})$

$\cos(\frac{\pi}{3}) = \frac{1}{2}$

(5) $\cos\left(-\frac{4\pi}{3}\right) = -\frac{1}{2}$

(6) $\cos(\frac{11\pi}{2}) = \cos(\frac{3\pi}{2} + 4\pi) = 0$

(7) $\sin(-\frac{5\pi}{4}) = -\sin \frac{5\pi}{4} = \frac{\sqrt{2}}{2}$

(8) $\sin(-\frac{2\pi}{3}) = -\sin(\frac{2\pi}{3}) = -\frac{\sqrt{3}}{2}$

(10) $\cos(-\frac{8\pi}{3}) = \cos \frac{8\pi}{3} = \cos \frac{2\pi}{3} + 2\pi = -\frac{1}{2}$

(11) $\sin(-\frac{5\pi}{4}) = -\sin \frac{5\pi}{4} = \frac{\sqrt{2}}{2}$

(12) $\cos(\frac{7\pi}{3}) = \cos \frac{\pi}{3} + 2\pi = \frac{1}{2}$

lecture #3

Define:

$\tan x = \frac{\sin x}{\cos x}$ including Domain & Range of all 6 Trig fctns

$\sec x = \frac{1}{\cos x}$

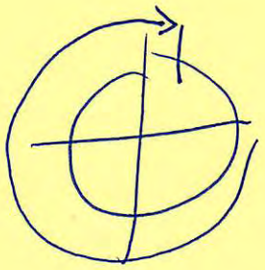
$\csc x = \frac{1}{\sin x}$

$\cot x = \frac{\cos x}{\sin x}$

$\tan \frac{25\pi}{3}$

$\tan 8\frac{1}{3}\pi$ ~~eight-thirds~~ four times around plus $\frac{\pi}{3}$ so

coord is $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ so $\tan \frac{\pi}{3} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$.

$\sec\left(-\frac{5\pi}{3}\right) =$ 

$\frac{1}{\frac{1}{2}} = 2$

HW Pg 53 1-8 Draw circles ♯ mark

Pg 59 1-11

Not enough time for students to hw on Thursday. Discuss 4th Lecture

1. Prove $\cos x$ is even fctn.

$f(x) = \cos x$

$f(-x) = \cos(-x)$

$= \cos(0 - x)$

$= \cos 0 \cos x + \sin 0 \sin x$

$= \cos x + 0$

$f(-x) = \cos x$

$f(-x) = f(x)$

also included Period & The Fctns.

2. $G(x) = \sin x$

$G(-x) = \sin(-x)$

$= \sin(0 - x)$

$= \sin 0 \cos x - \sin x \cos 0$

$0 \qquad\qquad 1$

$G(-x) = -\sin x$

$= -G(x)$ $\therefore$ Sin is odd fctn.

Some previous day

3. $f(x) = \cos x$

$D_F \quad x \in \mathbb{R}$

$R_f \quad -1 \le f(x) \le 1$

or $|f(x)| \le 1$

4. $G(x) = \sin x$

$D_G \quad x \in \mathbb{R}$

$R_G \quad -1 \le G(x) \le 1$

or $|G(x)| \le 1$

5. Define Tangent

$H(x) = \tan x = \dfrac{\sin x}{\cos x}$

$D_H \quad \cos x \overset{\text{set}}{=} 0$

$\frac{\pi}{2} + k\pi = \frac{(2k+1)\pi}{2}$

$D_H \quad x \notin \left\{\dfrac{(2k+1)\pi}{2}\right\}$ $\qquad R_H \quad \tan x \in \mathbb{R}$.

more ↓

6. is $\tan x$ odd or even? Prove it.

$H(x) = \tan x$

$H(-x) = \tan(-x)$

$= \dfrac{\sin(-x)}{\cos(-x)}$

$= \dfrac{-\sin x}{\cos x}$

$= -\tan x$

Need to include in last example our next type $\tan x = \frac{\sin x}{\cos x}$.

$H(-x) = -H(x)$ ∴ Tangent is odd fctn.

7. ~~Show~~ Cos

to show a fctn has a certain period,

show $f(x+a) = f(x)$

cos int has period 2π

$\cos(x+2\pi) = \cos x \cos 2\pi - \sin x \sin 2\pi$

$= \cos x$

$f(x+2\pi) = f(x)$ ∴ f has period 2π

8. show sine has period 2π

9. show tangent has period π

10. derive Tan Add

$$\tan(A \pm B) = \frac{\sin(A \pm B)}{\cos(A \pm B)} = \frac{\sin A \cos B \pm \sin B \cos A}{\cos A \cos B \mp \sin A \sin B}$$

$$\frac{\frac{1}{\cos A \cos B}}{\frac{1}{\cos A \cos B}}$$

$$\frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

HW Quiet Secrets Part IV

4th Ed

$$\tan(A \pm B) =$$

$$\frac{\sin(A \pm B)}{\cos(A \pm B)} =$$

$$\left(\frac{\sin A \cos B \pm \sin B \cos A}{\cos A \cos B \mp \sin A \sin B} \right) \frac{\frac{1}{\cos A \cos B}}{\frac{1}{\cos A \cos B}}$$

$$\frac{\frac{\sin A \cancel{\cos B}}{\cancel{\cos A}\cancel{\cos B}} \pm \frac{\sin B \cancel{\cos A}}{\cancel{\cos A}\cos B}}{\frac{\cos A \cos B}{\cos A \cos B} \mp \frac{\sin A \sin B}{\cos A \cos B}}$$

$$= \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

is tangent odd, even or neither?

$\tan(-x) \stackrel{?}{=} \tan x$ or $-\tan x$

$$\tan(0 - x) = \frac{\tan 0 - \tan x}{1 + \tan 0 \tan x}$$

$$= -\tan x$$

$\therefore \tan$ is an odd fctn.

↓

4 Review (cont)

Show that tan has period π

$\tan(x+\pi) \stackrel{?}{=} \tan x$

$$\frac{\tan x + \tan \pi}{1 - \tan x \tan \pi}$$

$\tan x = \tan x$ so tan has period π.

* to show a fctn has period a you must show that $fctn(x+a) = fctn(x)$.

Given $\tan x = 3$ $\quad \pi < x < \frac{3\pi}{2}$

find $\sin 2x$ ....

$$\frac{\sin x}{\cos x} = 3$$

$\sin x = 3\cos x$

$\sin^2 x = 9\cos^2 x$

$\sin^2 x = 9(1 - \sin^2 x)$

$\sin^2 x = 9 - 9\sin^2 x$

$10\sin^2 x = 9$

$\sin^2 x = \frac{9}{10}$

$\sin x = \pm \frac{3}{\sqrt{10}} = \pm \frac{\sqrt{30}}{10}$

But x is in 3rd quad so $-\frac{\sqrt{30}}{10}$

$s^2 + c^2 = 1$

$\frac{9}{10} + \cos^2 x = 1$

$\cos^2 x = \frac{1}{10}$

$\cos x = \pm \frac{\sqrt{10}}{10}$

3rd quad so $-\frac{\sqrt{10}}{10}$

Now ....

$\sin 2x =$

$2\sin x \cos x$

$2\left(\frac{-\sqrt{3}}{\sqrt{10}}\right)\left(-\frac{1}{\sqrt{10}}\right)$

$\frac{\sqrt{3}}{5}$

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2-11 Pg 86-87 Proving identities

Do ex 1+2 on board Do NOT Allow method of ex 2

H.W Read carefully Pg 86-87
Do Problems 1-8, 14.

Distinguish between a Proof & a verification.

A Proof must keep one side as given & change the other side to agree with unchanged side. A verification simply requires that the final line is obvious identity such as $1=1$ or $\cos x = \cos x$ or $\cos^2 x + \sin^2 x = 1$

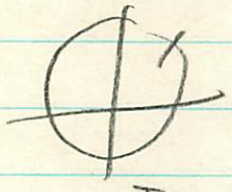
OR

Start w/ known identity & end

It entirely with original statement.

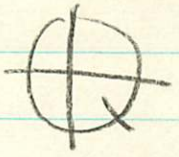
(21) $x \in$ IV, I

a) $\tan x = 1$



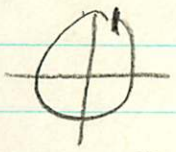
$x = \frac{\pi}{4}$

b) $\tan x = -1$



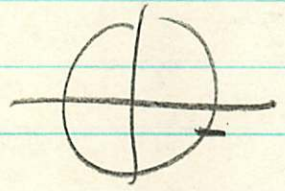
$x = \frac{-\pi}{4}$

c) $\tan x > \sqrt{3}$



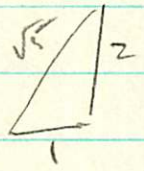
$x = \frac{\pi}{3}$

d) $\tan x = -\frac{1}{\sqrt{3}} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$



$x = -\frac{\pi}{6}$

(22) $\tan x = 2$



a) 1ST Quad

$s^2 + c^2 = 1$

$t^2 + 1 = \sec^2$

$4 + 1 = \sec^2$

$\cos x = \frac{\sqrt{5}}{5}$

$\frac{1}{5} + \sin^2 x = 1$

$\sin x = \pm \frac{4\sqrt{5}}{5}$

$\sin x = \frac{4\sqrt{5}}{5}$

1ST Quad.

(7) $\cos 2x + 2\sin^2 x = 1$

$\cos^2 x - \sin^2 x + 2\sin^2 x = 1$

$\cos^2 x + \sin^2 x = 1$

$x \in \mathbb{R}$

(8) $\sin 2x \cos x - \cos 2x \sin x = \sin x$

$\sin A - B$

$\sin(2x - x) =$

$\sin x =$

$x \in \mathbb{R}$

(9) $(\sin x + \cos x)^2 = 1 + \sin 2x$

$= \sin^2 x + \cos^2 x + 2\sin x \cos x$

$= (\sin x + \cos x)^2$

$x \in \mathbb{R}$

(10) $(\sin x - \cos x)^2 = 1 - \sin 2x$

$= \sin^2 x + \cos^2 x - 2\sin x \cos x$

$= (\sin x - \cos x)^2$

$x \in \mathbb{R}$

(11) $\cos^4 x - \sin^4 x = \cos 2x$

$(\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x) =$

$1 \qquad \cos 2x = \cos 2x$

$x \in \mathbb{R}$

(12) $2\sin\frac{x}{2}\cos\frac{x}{2} = \sin x$

$= \sin\frac{x}{2} + \frac{x}{2}$

$= 2\sin\frac{x}{2}\cos\frac{x}{2}$

(13) $\sin 3x = 3\sin x - 4\sin^3 x$

$\sin(2x+x)$

$\sin 2x \cos x + \sin x \cos 2x$

$2\sin x \cos^2 x + \sin x(\cos^2 x - \sin^2 x)$

$3\sin x \cos^2 x - \sin^3 x =$

$3\sin x(1-\sin^2 x) + \sin^3 x =$

$4\sin x - 3\sin^3 x =$

$x \in \mathbb{R}$

(14) $\cos 3x = 4\cos^3 x - 3\cos x$

$\cos(2x+x) =$

$\cos 2x \cos x - \sin 2x \sin x$

$\cos^3 x - \sin^2 x \cos x - 2\sin^2 x \cos x =$

$\cos^3 x - 3\sin^2 x \cos x =$

$\cos^3 x - 3\cos x + 3\cos^3 x =$

$4\cos^3 x - 3\cos x =$

$x \in \mathbb{R}$

(15) $\sin 4x = 4\sin x \cos x - 8\sin^3 x \cos x$

$2\sin 2x \cos 2x =$

$4\sin x \cos x(\cos^2 x - \sin^2 x) =$

$4\sin x \cos x(1 - 2\sin^2 x) =$

$4\sin x \cos x - 8\sin^3 x \cos x$

$x \in \mathbb{R}$

(16) $\cos 4x = 1 - 8\sin^2 x \cos^2 x$

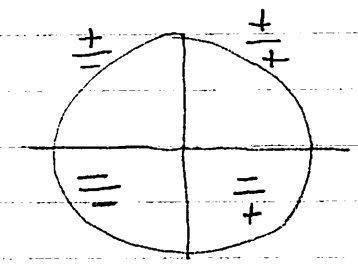
$\cos^2 2x - \sin^2 2x =$

$(\cos^2 x - \sin^2 x)^2 - 4\sin^2 x \cos^2 x =$

$\cos^4 x - 2\sin^2 x \cos^2 x + \sin^4 x =$

$\cos^2 x(1-\sin^2 x) - 2\sin^2 x \cos^2 x + \sin^2 x(1-\cos^2 x) =$

2-9

① 

$\Rightarrow$ $\tan > 0$ in Quad 1, 3
$\tan < 0$ in Quad 2, 4

② $\tan x = \frac{\sin x}{\cos x}$ $\tan$ is undefined at $\cos = 0$
or $\frac{\pi}{2} + k\pi = x$

③ $\tan \frac{3\pi}{4} = \tan(\pi - \frac{\pi}{4}) = -\tan \frac{\pi}{4} = \frac{-\sin \frac{\pi}{4}}{\cos \frac{\pi}{4}} =$

$\frac{-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = -1$

④ $\tan \frac{2\pi}{3} = \tan(\pi - \frac{\pi}{3}) = -\tan \frac{\pi}{3} = -\frac{\sin \frac{\pi}{3}}{\cos \frac{\pi}{3}} =$

$\frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = -\sqrt{3}$

⑤ $\tan \frac{5\pi}{6} = \tan(\pi - \frac{\pi}{6}) = -\tan \frac{\pi}{6} = \frac{-\sin \frac{\pi}{6}}{\cos \frac{\pi}{6}} =$

$\frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{\sqrt{3}}{3}$

⑥ $\tan \frac{7\pi}{6} = \tan(\pi + \frac{\pi}{6}) = \tan \frac{\pi}{6} = \frac{\sqrt{3}}{3}$

⑦ $\tan(-\frac{4\pi}{3}) = -\tan(\frac{4\pi}{3}) = -\tan(\pi + \frac{\pi}{3}) =$
$-\tan \frac{\pi}{3} = -\sqrt{3}$

⑧ $\tan(-\frac{5\pi}{4}) = -\tan \frac{5\pi}{4} = -\tan(\pi + \frac{\pi}{4}) =$

$\tan \frac{\pi}{4} = -1$

(9) $\tan \frac{29\pi}{6} = \tan\left(4\pi + \frac{5\pi}{6}\right) = \tan \frac{5\pi}{6} = -\frac{\sqrt{3}}{3}$

(10) $\tan \frac{19\pi}{3} = \tan\left(6\pi + \frac{\pi}{3}\right) = \tan \frac{\pi}{3} = \sqrt{3}$

(11) $\left(\frac{4}{5}\right)^2 + \cos^2 x = 1$ 1st Quad

$\frac{16}{25} + \cos^2 x = 1$

$\cos^2 x = \frac{9}{25}$

$\cos = \frac{3}{5}$

$\tan x = \frac{\frac{4}{5}}{\frac{3}{5}} = \frac{4}{3}$

(12) $\sin^2 x + \frac{25}{169} = 1$

$\sin^2 x = \frac{144}{169}$

$\sin x = \frac{12}{13}$ 1st Quad

$\tan x = \frac{\frac{12}{13}}{\frac{5}{13}} = \frac{12}{5}$

(13) $\sin > 0 \quad \cos < 0$

$\sin^2 x + \frac{25}{13} = 1$

$\sin x = \frac{12}{13}$

$\tan x = -\frac{12}{5}$

(14) $\cos < 0 \quad \frac{9}{25} + \cos^2 x = 1$

$\cos^2 x = \frac{16}{25}$

$\cos x = \frac{4}{5}$

$\tan x = -\frac{3}{5} \cdot -\frac{5}{4} = \frac{3}{4}$

(15) $\tan(x_2 + x_1) \quad \sin x_2 = \frac{1}{2} \quad \cos x_1 = \frac{\sqrt{3}}{2}$

$0 < x_2 < \frac{\pi}{2}$

$0 < x_1 < \frac{\pi}{2}$

$x_2 = \frac{\pi}{6} \quad x_1 = \frac{\pi}{6}$

$\tan \frac{\pi}{3} = \frac{\sin \frac{\pi}{3}}{\cos \frac{\pi}{3}} = \frac{\sqrt{3}}{2} \cdot \frac{2}{1} = \sqrt{3}$

(16) $\tan(x_2 - x_1)\cos x_2 = -\frac{\sqrt{2}}{2} \quad \sin x_1 = \frac{\sqrt{3}}{2}$

$x_2 = \frac{3\pi}{4} \qquad \frac{\pi}{2} < x_2 < \pi$

$x_1 = \frac{\pi}{3} \qquad 0 < x_1 < \frac{\pi}{2}$

$$\tan\left(\frac{3\pi}{4} - \frac{\pi}{3}\right) = \frac{\sin\frac{3\pi}{4}\cos\frac{\pi}{3} - \sin\frac{\pi}{3}\cos\frac{3\pi}{4}}{\cos\frac{3\pi}{4}\cos\frac{\pi}{3} + \sin\frac{3\pi}{4}\sin\frac{\pi}{3}} =$$

$$\frac{\frac{\sqrt{2}}{2}\cdot\frac{1}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}\cdot\frac{1}{2} + \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2}} = \frac{\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}}{\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}} = \frac{\sqrt{2}+\sqrt{6}}{4}\cdot\frac{4}{\sqrt{6}-\sqrt{2}} =$$

$$\frac{\sqrt{2}+\sqrt{6}}{\sqrt{6}-\sqrt{2}} = \frac{\sqrt{2}(1+\sqrt{3})}{\sqrt{2}(\sqrt{3}-1)} = \frac{\sqrt{3}}{\sqrt{3}-1}\,\frac{\sqrt{3}+1}{\sqrt{3}+1} + \frac{1}{\sqrt{3}-1}\,\frac{\sqrt{3}+1}{\sqrt{3}+1} \Rightarrow$$

$$\frac{3+\sqrt{3}}{3-1} + \frac{\sqrt{3}+1}{3-1} = \frac{3+\sqrt{3}+\sqrt{3}+1}{2} = \frac{4+2\sqrt{3}}{2} = 2+\sqrt{3}.$$

(17) $\tan(x-\pi) = \tan x$

$$\tan(x-\pi) = \frac{\sin x\cos\pi - \sin\pi\cos x}{\cos x\cos\pi + \sin x\sin\pi} = \frac{-\sin x}{-\cos x} = \tan x$$

$\cos x \neq 0$

(18) $\tan 2x = \frac{2\tan x}{1-\tan^2 x}$

$$\tan 2x = \tan(x+x) = \frac{\tan x + \tan x}{1 - \tan x\tan x} =$$

$\tan^2 x \neq 1$

$$\frac{2\tan x}{1-\tan^2 x}$$

$\sin x \neq 0$
$x \neq k\pi$ 982

(19) $\tan\frac{x}{2} = \frac{1-\cos x}{\sin x}$

$\sin\left(\frac{x}{2}+\frac{x}{2}\right) = 2\sin\frac{x}{2}\cos\frac{x}{2} = \sin x.$

$= \frac{1-\cos x}{2\sin\frac{x}{2}\cos\frac{x}{2}}$ $\qquad \cos\left(\frac{x}{2}+\frac{x}{2}\right) = \cos^2\frac{x}{2} - \sin^2\frac{x}{2}$

$\cos x =$

$= \frac{1-\left[\cos^2\frac{x}{2} - \sin^2\frac{x}{2}\right)}{2\sin\frac{x}{2}\cos\frac{x}{2}}$

$= \frac{1-\left[1-\sin^2\frac{x}{2} - \sin^2\frac{x}{2}\right]}{2\sin\frac{x}{2}\cos\frac{x}{2}}$

$= \frac{1-1+2\sin^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}$

$= \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}$

$= \tan\frac{x}{2}, \qquad x \neq k\pi$

(20) $\tan(x_2 - x_1) = \frac{\tan x_2 - \tan x_1}{1+\tan x_2 \tan x_1}$

$= \frac{\sin(x_2-x_1)}{\cos(x_2-x_1)} = \frac{\sin x_2\cos x_1 - \sin x_1\cos x_2}{\cos x_2\cos x_1 + \sin x_2\sin x_1}$ $\quad \frac{\frac{1}{\cos x_1\cos x_2}}{\frac{1}{\cos x_1\cos x_2}}$

$= \frac{\tan x_2 - \tan x_1}{1 - \tan x_2 \tan x_1}.$

82

21 $\tan(2\pi - x) = -\tan x$

$$\frac{\sin(-(x-2\pi))}{\cos(-(x-2\pi))} = \frac{-\sin(x-2\pi)}{\cos(x-2\pi} = \frac{-\sin x}{\cos x} = -\tan x$$

(22) $\tan\left(x+\frac{\pi}{4}\right) = \frac{1+\tan x}{1-\tan x}$

$$\sin\left(x+\frac{\pi}{4}\right) = \frac{\sqrt{2}\sin x}{2} + \frac{\sqrt{2}\cos x}{2} \qquad \frac{1}{\sqrt{2}\cos x}$$

$$\cos\left(x+\frac{\pi}{4}\right) \quad \frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x, \quad \frac{1}{\cos x}$$

$$\frac{\frac{\sin x}{\cos x}+1}{1-\frac{\sin x}{\cos x}}$$

$$\frac{\tan x + 1}{1-\tan x}$$

$x \neq \sum \frac{(2k+1)\pi}{2}$

(23) $\tan x = \pm\sqrt{\frac{1-\cos 2x}{1+\cos 2x}}$

27 $\tan x + \tan^2 x \tan 2x = \tan 2x - \tan x$

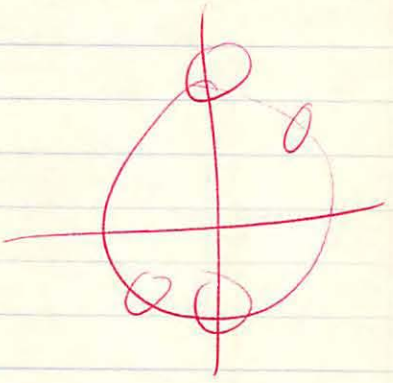
$$\tan x + \tan^2 x \left(\frac{2\tan x}{1-\tan^2 x}\right) =$$

$$\frac{\tan x - \tan^3 x + 2\tan^3 x}{1-\tan^2 x} =$$

$$\frac{\tan x + \tan^3 x}{1-\tan^2 x} = \frac{2\tan x - \tan x + \tan^3 x}{1-\tan^2 x} =$$

$$\frac{2\tan x}{1-\tan^2 x} - \frac{\tan x(1-\tan^2 x)}{1-\tan^2 x}$$

$\tan 2x - \tan x$ $\qquad x \neq \left\{\frac{(2k+1)\pi}{2}, \frac{(2k+1)\pi}{4}\right\}$



(20) $x_1+x_2+x_3=\pi$ WTP $\tan x_1+\tan x_2+\tan x_3=$

$\tan x_1 \tan x_2 \tan x_3$

$$\tan(x_1+x_2)=\frac{\tan x_1+\tan x_2}{1-\tan x_1 \tan x_2}$$

$$\tan(x_1+x_2)(1-\tan x_1 \tan x_2)=\tan x_1+\tan x_2$$

$$\tan(x_1+x_2)(1-\tan x_1 \tan x_2)+\tan x_3=\tan x_1+\tan x_2+\tan x_3$$

$$\tan(x_1+x_2)(1-\tan x_1 \tan x_2)+\tan x_3=$$

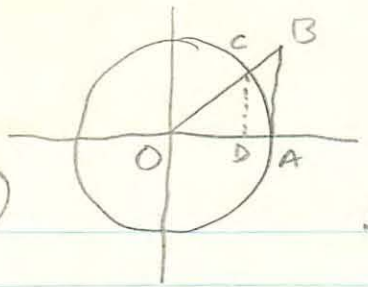
$$\tan(\pi-x_3)(1-\tan x_1 \tan x_2)+\tan x_3=$$

$$\frac{\tan\pi-\tan x_3}{1+\tan\pi\tan x_3}(\quad)+\tan x_3=$$

$$-\tan x_3(1-\tan x_1\tan x_2)+\tan x_3=$$

$$-\tan x_3+\tan x_1\tan x_2\tan x_3+\tan x_3=$$

$$\tan x_1\tan x_2\tan x_3=\tan x_1+\tan x_2+\tan x_3$$



(29) $\Delta COD \sim \Delta BOA$, AA

$$\frac{\overline{OC}}{\overline{OB}} = \frac{\overline{CD}}{\overline{BA}} = \frac{\overline{OD}}{\overline{OA}} \qquad OA = OC = 1 \text{ UNIT circle}$$

$$\therefore \quad \frac{1}{\overline{OB}} = \frac{\overline{CD}}{\overline{BA}} = \frac{\overline{OD}}{1} \qquad OD = \cos x \quad CD = \sin x,$$

$$\frac{1}{\overline{OB}} = \frac{\sin x}{\overline{BA}} = \cos x.$$

$$\overline{BA} = \tan x$$

(30) $x_1 + x_2 + x_3 = \pi$ WTS $\tan x_1 + \tan x_2 + \tan x_3 = \tan x_1 \tan x_2 \tan x_3$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\frac{\tan x_1 + \tan x_2}{1 - \tan x_1 \tan x_2} = \tan(x_1 + x_2)$$

add $\tan x_3$ both sides.

$$\tan x_3 + \tan x_1 + \tan x_2 = \tan(x_1 + x_2)(1 - \tan x_1 \tan x_2) + \tan x_3$$

$$\tan(x_1 + x_2)(1 - \tan x_1 \tan x_2) + \tan x_3$$

$$\tan(\pi - x_3)(1 - \tan x_1 \tan x_2) + \tan x_3$$

$$-\tan x_3 \,(1 - \tan x_1 \tan x_2) + \tan x_3$$

$$-\tan x_3 + \tan x_1 \tan x_2 \tan x_3 + \tan x_3$$

$$\tan x_1 \tan x_2 \tan x_3 = \tan x_1 \tan x_2 \tan x_3$$

2-10 Sec, Csc, Cot

We have Reciprical fctns of our 3 well known circular fcts. Just as x has a Reciprical $\frac{1}{x}$.

Secant $= \frac{1}{\cos x} = \sec x$ $\quad x \neq \frac{(2k+1)\pi}{2}$

Cosecant $= \frac{1}{\sin x} = \csc x$ $\quad x \neq k\pi$

Cotangent $= \frac{\cos x}{\sin x} = \cot x$ $\quad x \neq k\pi$

$\cot x \equiv \frac{\cos x}{\sin x}$ $\quad x \neq k\pi$ If $\cot x$ is defined as $\frac{1}{\tan x}$ then restrictions are $x \in \{k\pi, \frac{(2k+1)\pi}{2}\}$ Since $\tan x = 0$ is <u>Also</u> not defined

Notice the reversal of the limiting factors ie. $\sec x$ not defined for $\cos x = 0$ or $x = \frac{\pi}{2} + k\pi$ $k \in J$

H.W. Pg 85 5-11 9-19 odd

Pg 85

⑤ $\sec x$, $\sin = \frac{-3}{5}$ $\cos x > 0$ $\frac{9}{25} + \cos^2 x = 1$ $\cos^2 x = \frac{16}{25}$

$\cos x = \frac{4}{5}$ ∴ $\sec x = \frac{5}{4}$

⑥ $\csc x$ $\cos x = \frac{4}{5}$ $\sin < 0$ $\frac{16}{25} + \sin^2 x = 1$ $\sin^2 x = \frac{9}{25}$

$\sin x = \frac{-3}{5}$ $\csc x = \frac{-5}{3}$

⑦ $\cot x$ $\sin x = \frac{5}{13}$ $\cos x < 0$ $\frac{25}{169} + \cos^2 x = 1$ $\cos^2 x = \frac{144}{169}$

$\cot x = \frac{\frac{-12}{13}}{\frac{5}{13}} = \frac{-12}{5}$ $\cos x = \frac{-12}{13}$

⑧ $\cot x$ when $\cos = \frac{1}{\sqrt{2}}$ + $\sin > 0$ $\frac{1}{2} + \sin^2 x = 1$

$\sin^2 x = \frac{1}{2}$ $\sin = \frac{1}{\sqrt{2}}$

$\cot x = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 1$

⑨ $\cot x = \frac{1}{\tan x} = \frac{1}{\frac{\sin x}{\cos x}} \cdot \frac{\frac{\cos x}{\sin x}}{\frac{\cos x}{\sin x}} = \frac{\cos x}{\sin x} = \cot x$.

⑩ $\tan^2 x + 1 = \sec^2 x$ $\sin^2 x + \cos^2 x = 1$

$\frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$ $\tan^2 x + 1 = \sec^2 x$

⑪ $1 + \cot^2 x = \csc^2 x$

$\frac{\sin^2 x + \cos^2 x}{\sin^2 x} = 1$

$1 + \cot^2 x = \csc^2 x$

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① $\sin x \sec x = \tan x \quad \frac{\sin x}{\cos x} = \tan x \quad \cos x \neq 0$

② $\sin x + \cos x \cot x = \csc x$

$\csc x \neq 0$

$$\frac{\sin x}{\csc x} + \frac{\cos x \cot x}{\csc x} = 1$$

$$\sin^2 x + \frac{\cos x \cos x}{\sin x} \cdot \frac{1}{\csc x} = 1$$

$$\sin^2 x + \cos x \frac{\cos x}{\sin x} \cdot \sin x = 1$$

$$\sin^2 x + \cos^2 x = 1$$

③ $\cos x \csc x = \cot x \quad \frac{\cos x}{\sin x} = \cot x \quad \sin x \neq 0$

④ $\tan x (\sin x + \cot x \cos x) = \sec x$

$$\left(\frac{\sin^2 x}{\cos x} + \cos x = \frac{1}{\cos x}\right) \cos x$$

$\cos x \neq 0$

$$\sin^2 x + \cos^2 x = 1$$

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(5) $\sec^2 x(1-\sin^2 x)=1$ $\quad \cos^2 x \neq 0$

$\sec^2 x \cdot \cos^2 x = 1$

$1 = 1$ $\quad x \neq \left\{\frac{(2k+1)\pi}{2}\right\}$

(6) $2\cos^2 x - \sin^2 x + 1 = 3\cos^2 x$ $\quad \cos^2 x + \cos^2 x + 1$

$2\cos^2 x + \cos^2 x = 3\cos^2 x$

$3\cos^2 x = 3\cos^2 x$ $\quad$ No Restr.

(7) $\csc^2 x(1-\cos^2 x)=1$

$\frac{1}{\sin^2 x} \cdot \sin^2 x = 1$ $\quad x \neq \{k\pi\}$

$1 = 1$

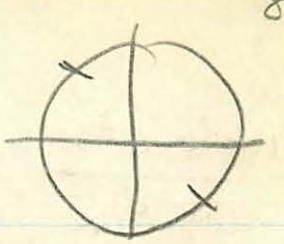
(8) $\sin x \tan x + \cos x = \sec x$

$\frac{\sin^2 x}{\cos x} + \cos x = \sec x$

$\frac{\sin^2 x + \cos^2 x}{\cos x} =$ $\quad x \neq \left\{\frac{(2k+1)\pi}{2}\right\}$

$\frac{1}{\cos x} =$

$\sec x = \sec x$



(14) $\dfrac{1+2\sin x\cos x}{\sin x+\cos x} = \sin x+\cos x$

$\dfrac{\sin^2 x+2\sin x\cos x+\cos x}{\sin x+\cos x}$

$\dfrac{(\sin x+\cos x)^2}{\sin x+\cos x}$

$\sin x+\cos x = \sin x+\cos x$

$\sin x+\cos x = 0$

$\sin x = -\cos x$

1st place is $\frac{3\pi}{4}$

Next is π more so

$\frac{3\pi}{4}+k\pi \Rightarrow$

$\frac{3\pi+4k\pi}{4} = \frac{(3+4k)\pi}{4}$

$x \notin \left\{\frac{(3+4k)\pi}{4}\right\}$

(15) $\sin 4x\cos x-\cos 4x\sin x=\sin 3x$

Sin Subtract form.

$\sin(4x-x) =$

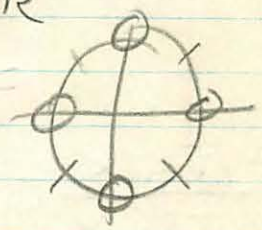
$\sin 3x = \sin 3x$ $\quad x\in\mathbb{R}$

(16) $\dfrac{\sin x\cos x}{1-2\sin^2 x} = \dfrac{1}{\cot x-\tan x}$

$\dfrac{1}{\frac{\cos x}{\sin x}-\frac{\sin x}{\cos x}}$

$\dfrac{1}{\frac{\cos^2 x-\sin^2 x}{\sin x\cos x}}$

$\dfrac{\sin x\cos x}{1-2\sin^2 x}$



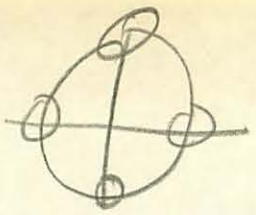
$\cot x = \tan x$

if $\sin^2 x = \cos^2 x$

$\therefore x\in\mathbb{R}$

$x \notin \left\{\frac{k\pi}{4}\right\}$

2nd version follows ↓



(17) $\dfrac{1+\tan^2 x}{\tan^2 x} = \csc^2 x$

$\dfrac{1+\dfrac{\sin^2 x}{\cos^2 x}}{\dfrac{\sin^2 x}{\cos^2 x}} = \dfrac{\overbrace{\cos^2 x+\sin^2 x}^{1}}{\cos^2 x} \cdot \dfrac{\cos^2 x}{\sin^2 x} = \dfrac{1}{\sin^2 x}$

$x \in \mathbb{R} \setminus \left\{\dfrac{k\pi}{2}\right\}$ $\csc^2 x$

(18) $\cos^4 x - \sin^4 x = \cos^2 x - \sin^2 x$

$\underbrace{(\cos^2 x+\sin^2 x)}_{1}(\cos^2 x-\sin^2 x) = \cos^2 x-\sin^2 x \quad x \in \mathbb{R}$

$\cos^2 x-\sin^2 x = \cos^2 x-\sin^2 x$

(19) $(1+\sin x)^2 = 2(1+\sin x)-\cos^2 x$

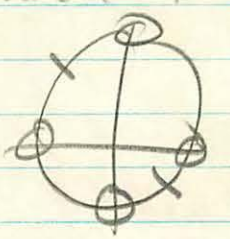
$= 2+2\sin x-(1-\sin^2 x)$

$= 1+2\sin x+\sin^2 x$

$= (1+\sin x)^2 \quad x \in \mathbb{R}$

(20) $\dfrac{1-\tan x}{1+\tan x} = \dfrac{\cot x-1}{\cot x+1}$ $\quad \tan x \neq -1$

$= \dfrac{\dfrac{\cos x-\sin x}{\sin x}\cdot\dfrac{1}{\sin x}}{\dfrac{\cos x+\sin x}{\sin x}\cdot\dfrac{1}{\cos x}}$



$\dfrac{1-\tan x}{1+\tan x} = \dfrac{1-\tan x}{1+\tan x}$ $\quad x \in \mathbb{R}$, $x \notin \left\{\dfrac{k\pi}{2}, \dfrac{3\pi}{4}+k\pi\right\}$

16) $\dfrac{\sin x \cos x}{1-2\sin^2 x} = \dfrac{1}{\cot x - \tan x}$

$\dfrac{\frac{1}{\sin x \cos x}}{\frac{1-2\sin^2 x}{\sin x \cos x}} = \dfrac{1}{\frac{1-\sin^2 x - \sin^2 x}{\sin x \cos x}} = \dfrac{1}{\frac{\cos^2 x - \sin^2 x}{\sin x \cos x}} =$

$\dfrac{1}{\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}} = \dfrac{1}{\cot x - \tan x}$

$\cot x - \tan x = 0$

$\cot x = \tan x$

$\dfrac{\cos x}{\sin x} = \dfrac{\sin x}{\cos x}$

$\cos^2 x = \sin^2 x$

$\cos^2 x - \sin^2 x = 0$

$1 - \sin^2 x - \sin^2 x = 0$

$1 - 2\sin^2 x = 0$ SAME AS

$2\sin^2 x - 1 = 0$

$2\sin^2 x = 1$

$\sin^2 x = \frac{1}{2}$

$\sin x = \pm \frac{\sqrt{2}}{2}$

$x \in \left\{\frac{\pi}{4} + \frac{k\pi}{2}\right\}$

LASTLY

$\sin x \cos x = 0$

$\sin x = 0$ or $\cos x = 0$ $\frac{k\pi}{2}$

combining restrictions on x

$\frac{\pi}{4} + \frac{k\pi}{2}$ and $\frac{k\pi}{2} \Rightarrow \frac{k\pi}{4}$ $\therefore$ $x \in \left\{\frac{k\pi}{4}\right\}$,

17) $\dfrac{1+\tan^2 x}{\tan^2 x} = \csc^2 x$

$$\frac{\frac{\cos^2 x + \sin^2 x}{\cos^2 x}}{\frac{\sin^2 x}{\cos^2 x}} = \frac{\frac{1}{\cos^2 x}}{\frac{\sin^2 x}{\cos^2 x}} = \frac{1}{\sin^2 x} = \csc^2 x.$$

$\tan^2 x \neq 0$

$\sin x \neq 0 \Rightarrow x \neq \{k\pi\}$

$\cos x \neq 0 \Rightarrow x \neq \left\{\frac{(2k+1)\pi}{2}\right\}$ $\Bigg\}$ $x \notin \left\{\frac{k\pi}{2}\right\}$

18) $\cos^4 x - \sin^4 x = \cos^2 x - \sin^2 x \qquad x \in \mathbb{R}$

$(\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x) =$

$(\cos^2 x - \sin^2 x)(1) = \cos^2 x - \sin^2 x.$

19) $(1+\sin x)^2 = 2(1+\sin x) - \cos^2 x$

$= 2(1+\sin x) - 1 + \sin^2 x$

$= 1 + 2\sin x + \sin^2 x$

$= (1+\sin x)^2 \qquad x \in \mathbb{R}$

20) $\dfrac{1-\tan x}{1+\tan x} = \dfrac{\cot x - 1}{\cot x + 1}$

$\tan x \Rightarrow \cos x \neq 0 \quad x \notin \left\{\frac{(2k+1)\pi}{2}\right\}$

$1+\tan x \neq 0$

$\tan x \neq -1$

$\sin x \neq -\cos x$

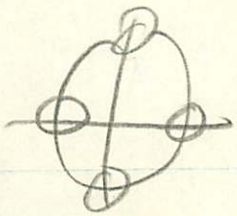
$x \notin \left\{\frac{3\pi}{4} + k\pi\right\}$

$1+\cot x \neq 0$

$\cot x \neq -1$ SAME.

$$= \frac{\frac{\cos x}{\sin x} - 1}{\frac{\cos x}{\sin x} + 1} = \frac{\frac{\cos x - \sin x}{\sin x}}{\frac{\cos x + \sin x}{\sin x}} \sin x = \frac{\cos x - \sin x}{\cos x + \sin x} = \frac{1 - \frac{\sin x}{\cos x}}{1 + \frac{\sin x}{\cos x}} = \frac{1-\tan x}{1+\tan x}$$

LASTLY $\sin x \neq 0 \quad x \notin k\pi$ Combining $x \notin \left\{\frac{k\pi}{2}, \frac{3\pi}{4} + k\pi\right\}$



Again. Must PS.

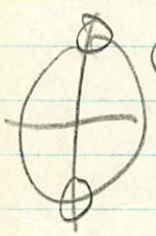
(21) $\dfrac{1-\sin^2 x}{1-\cos^2 x} = \cot^2 x$

$\dfrac{\cos^2 x}{\sin^2 x} \Rightarrow \cot^2 x = \cot^2 x \quad x \notin \left\{\dfrac{k\pi}{2}\right\}$

(22) $\dfrac{\sin(x-y)}{\sin(x+y)} = \dfrac{\tan x - \tan y}{\tan x + \tan y}$

$\dfrac{\sin x \cos y - \sin y \cos x}{\sin x \cos y + \sin x \cos x}$ $\quad \dfrac{\frac{1}{\cos x \cos y}}{\frac{1}{\cos x \cos y}}$

$\dfrac{\tan x - \tan y}{\tan x + \tan y}$



$x + y \neq 0 \qquad \dfrac{\sin x}{\cos x} \neq -\dfrac{\sin y}{\cos y}$

$\cos x \neq 0 \therefore x$

$x, y \notin \left\{\dfrac{(2k+1)\pi}{2}\right\}$

$x + y \neq k\pi$

21) $\frac{1-\sin^2 x}{1-\cos^2 x} = \cot^2 x$ $\quad 1-\cos^2 x \neq 0$

$\cos^2 x \neq 1$

$\cos x \neq \pm 1 \quad x \notin \{k\pi\}$.

$\frac{\cos^2 x}{\sin^2 x} = \cot^2 x$ $\quad \cot^2 x \Rightarrow \sin x \neq 0$ same.

22) $\frac{\sin(x-y)}{\sin(x+y)} = \frac{\tan x - \tan y}{\tan x + \tan y}$

$\frac{\sin x \cos y - \sin y \cos x}{\sin x \cos y + \sin y \cos x}$ $\quad \frac{\frac{1}{\cos x \cos y}}{\frac{1}{\cos x \cos y}}$

$\frac{\frac{\sin x}{\cos x} - \frac{\sin y}{\cos y}}{\frac{\sin x}{\cos x} + \frac{\sin y}{\cos y}} = \frac{\tan x - \tan y}{\tan x + \tan y}$.

$\sin(x+y) \neq 0 \Rightarrow x+y = z \notin \{k\pi\}$:

$\cos x \neq 0 \quad x \neq \frac{(2k+1)\pi}{2}$

$\cos y \neq 0 \quad y \neq \left\{\frac{(2k+1)\pi}{2}\right\}$

$\tan x + \tan y \neq 0$

$\tan x \neq -\tan y$

$\tan x \neq 0 \Rightarrow$

$\sin x \neq 0 \quad x \notin \{k\pi\}$

$\tan y \neq 0 \Rightarrow$

$\sin y \neq 0 \quad x \notin \{k\pi\}$.

$x = \frac{\pi}{4}$ and $y = \frac{3\pi}{4}$

If both $x \in \frac{(2k+1)\pi}{4}$ and $y \in \frac{(2k-1)\pi}{4}$ or vice versa then undefined as well.

24) $\cos x - \tan y \sin x = \sec y \cos(x+y)$

$= \frac{1}{\cos y}(\cos x \cos y - \sin x \sin y)$

$= \cos x - \tan y \sin x$

$x \in \mathbb{R}$

$\cos y \neq 0 \Rightarrow y \notin \left\{\frac{(2k+1)\pi}{2}\right\}$

26) $\frac{1-\cos 2x}{\sin 2x} = \tan x$

$\frac{1-\cos(x+x)}{\sin(x+x)} = \frac{1-\cos^2 x + \sin^2 x}{2\sin x \cos x} = \frac{2\sin^2 x}{2\sin x \cos x} = \tan x$

$\sin 2x \neq 0 \Rightarrow \sin(x+x) \neq 0 \Rightarrow 2\sin x \cos x \neq 0$

$\sin x \neq 0 \Rightarrow x \notin \{k\pi\}$

$\cos x \neq 0 \Rightarrow x \notin \left\{\frac{(2k+1)\pi}{2}\right\}$

$\tan x \Rightarrow \cos x \neq 0$ Same $x \notin \left\{\frac{k\pi}{2}\right\}$

27) $\tan 3x - \tan x = 2\sin x \sec 3x$

$\frac{\sin 3x}{\cos 3x} - \frac{\sin x}{\cos x} =$

$\frac{\cos x \sin 3x - \sin x \cos 3x}{\cos 3x \cos x} = \frac{\sin(3x-x)}{\cos 3x \cos x} = \frac{\sin 2x}{\cos 3x \cos x} =$

$\frac{2\sin x \cos x}{\cos 3x \cos x} = 2\sin x \sec 3x$

$\cos 3x \neq 0 \Rightarrow 3x \notin \left\{\frac{(2k+1)\pi}{2}\right\}$

$x \notin \left\{\frac{(2k+1)\pi}{6}\right\}$

$\cos x \neq 0 \Rightarrow x \notin \left\{\frac{(2k+1)\pi}{2}\right\}$

But this $\frac{\pi}{2}$ is when k is 1 so this covers both cases,

28 $\sin 3x + \sin x = 2\sin 2x \cos x$

$\sin(2x + x) + \sin x$

$\sin 2x \cos x + \sin x \cos 2x + \sin x$

$\sin x(\cos^2 x - \sin^2 x) + \sin x$

$\sin x \cos^2 x - \sin^3 x + \sin x$

$\sin x \cos^2 x + \sin x(1 - \sin^2 x)$

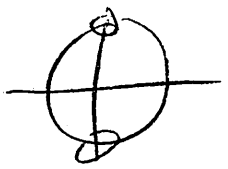
$\sin x \cos^2 x + \sin x \cos^2 x$

$2\sin x \cos^2 x$

$2\sin x \cos x \cos x$

$+ \quad \sin 2x \cos x$

$2\sin 2x \cos x$



$\cos 3x \neq 0$

$3x \notin \frac{(2k+1)\pi}{2}$ $\frac{3\pi}{6}$

(27) $\tan 3x - \tan x = 2\sin x \sec 3x$

$\frac{\sin 3x}{\cos 3x} - \frac{\sin x}{\cos x} =$

$x \notin \left\{\frac{(2k+1)\pi}{6}\right\}$

$\frac{\sin 3x \cos x - \cos 3x \sin x}{\cos 3x \cos x} =$

$\frac{\sin(3x-x)}{\cos 3x \cos x} = \frac{\sin 2x}{\cos 3x \cos x} = \frac{2\sin x \cos x}{\cos 3x \cos x} =$

$2\sin x \sec 3x = 2\sin x \sec 3x$

(28) $\sin 3x + \sin x = 2(\sin 2x \cos x)$

$\sin(2x+x) + \sin x =$

$\sin 2x \cos x + \cos 2x \sin x + \sin x$

$(\cos^2 x - \sin^2 x)\sin x + \sin x$

$(\cos x - \sin x)(\cos x + \sin x)\sin x + \sin x$

$(\quad)(\sin x \cos x + \sin^2 x) + \sin x$

$\cos^2 x \sin x - \sin^3 x + \sin x$

$(\cos^2 x + 1)\sin x - \sin^3 x$

$(\cos^2 x + 1 - \sin^2 x)\sin x$

$1 - \sin^2 + 1 - \sin^2 x$

$2(1 - \sin^2 x)\sin x$

also another version on next page.

28) $\sin 3x + \sin x = 2 \sin 2x \cos x$

$\sin(2x+x) + \sin x =$

$\sin 2x \cos x + \sin x \cos 2x + \sin x =$

$\sin 2x \cos x + \sin x (\cos^2 x - \sin^2 x) + \sin x =$

$\sin 2x \cos x + \sin x (2\cos^2 x - 1) + \sin x =$

$\sin 2x \cos x + 2 \sin x \cos^2 x - \sin x + \sin x =$

$\sin 2x \cos x + (\sin x \cos x + \sin x \cos x) \cos x =$

$\sin 2x \cos x + (\sin x + x) \cos x =$

$\sin 2x \cos x + \sin 2x \cos x =$

$2 \sin 2x \cos x, \qquad x \in \mathbb{R},$

(32) $\csc x - \cot x = \tan\frac{x}{2}$

$\frac{1}{\sin x} - \frac{\cos x}{\sin x}$ $\qquad \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}$

$\frac{1+\cos x}{1+\cos x}\ \frac{1-\cos x}{\sin x}$

$\frac{\sin x}{1+\cos x}$ $\qquad \begin{cases} \sin\left(\frac{x}{2}+\frac{x}{2}\right) = 2\sin\frac{x}{2}\cos\frac{x}{2} \\ \cos\left(\frac{x}{2}+\frac{x}{2}\right) = \cos^2\frac{x}{2} - \sin^2\frac{x}{2} \end{cases}$

$\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{1+\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}$

$\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}$

$\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \tan\frac{x}{2}$

90

$\cos x \cos 2x - \sin x \sin 2x$

(33) $\dfrac{\sin 4x - \sin 2x}{\sin 2x} = \dfrac{\cos 3x}{\cos x}$

$\dfrac{\sin(2x+2x) - \sin 2x}{\sin 2x}$

$= \dfrac{2\sin 2x \cos 2x - \sin 2x}{\sin 2x}$

$\sin 2x(2\cos 2x - 1)$

$2\cos 2x - 1$

$2(\cos^2 x - \sin^2 x) - \sin^2 x - \cos^2 x$

$\cos^2 x - 3\sin^2 x$

$\cos^2 x - \sin^2 x - 2\sin^2 x$

$\cos 2x - 2\sin^2 x$

$\sin(3x + x) - \sin 2x$

$\dfrac{\sin 3x \cos x + \sin x \cos 3x - \sin 2x}{\sin 2x}$

$-\sin 2x + \sin 2x \cos^2 x + \cos 2x \sin x \cos x +$

$-\sin 2x(1 - \cos^2 x)$

$-\sin 2x \sin^2 x + \cos 2x \sin x \cos x$

$+\sin x(\cos x \cos 2x - \sin 2x \sin x)$

$\sin x \cos 3x + \sin$

Quiz Solution.

① $\sin 2x = 2\sin x \cos x \qquad x \in \mathbb{R}$

$\sin(2x) = \sin(x+x)$

$= \sin x \cos x + \sin x \cos x$

$= 2\sin x \cos x$

② $\sin\left(\frac{3\pi}{2} - x\right) = -\cos x \qquad x \in \mathbb{R}$

$= \sin\frac{3\pi}{2}\cos x - \cos\frac{3\pi}{2}\sin x$ (0)

$= -\cos x$

③ $\sin(x+y) + \sin(x-y) = 2\sin x \sin y \qquad x, y \in \mathbb{R}$

$= \sin x \cos y + \sin y \cos x + \sin x \cos y - \sin y \cos x$

$= 2\sin x \sin y$

④ $\sin\left[(2k+1)\frac{\pi}{2} + x\right] = (-1)^k \cos x \qquad = 0 \ \forall k$

$\sin\frac{(2k+1)\pi}{2}\cos x + \sin x \cos\frac{(2k+1)\pi}{2}$ $\qquad x \in \mathbb{R}$

$\sin\frac{(2k+1)\pi}{2} = +1$ if $k \in 2M$

$\sin\frac{(2k+1)\pi}{2} = -1$ if $k \in 2M+1$,

if k is even $(-1)^{\text{even}} = +1$,

if k is odd $(-1)^{\text{odd}} = -1$,

original statement is true.

Extra Pblms

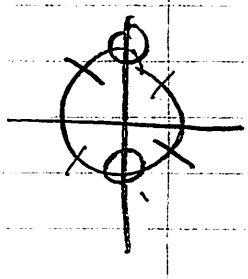
★ $\sin 3x = 3\sin x - 4\sin^3 x$

$\sin(2x+x) =$
$\sin 2x \cos x + \cos 2x \sin x$
$2\sin x \cos^2 x + (\cos^2 x - \sin^2 x)\sin x$
$2\sin x \cos^2 x + \sin x \cos^2 x - \sin^3 x$
$\cos^2 x (3\sin x) - \sin^3 x$
$(1 - \sin^2 x)$
$3\sin x - 3\sin^3 x - \sin^3 x$
$3\sin x - 4\sin^3 x \qquad x \in \mathbb{R}.$

★ $\left(\frac{1+\tan x}{1-\tan x}\right)^2 = \frac{1+\sin 2x}{1-\sin 2x}$

$= \frac{1 + 2\sin x \cos x}{1 - 2\sin x \cos x}$

$= \frac{\cos^2 x + \sin^2 x + 2\sin x \cos x}{\cos^2 x + \sin^2 x - 2\sin x \cos x}$



$= \frac{(\cos x + \sin x)^2}{(\cos x - \sin x)^2}$

$2\sin x \cos x = 1$
$\sin x \cos x = \frac{1}{2}$

$= \frac{\left(\cos x\left(1 + \frac{\sin x}{\cos x}\right)\right)^2}{\left(\cos x\left(1 - \frac{\sin x}{\cos x}\right)\right)^2}$

$\sin 2x = 1$

$= \frac{\cos^2 x\,(1+\tan x)^2}{\cos^2 x\,(1-\tan x)^2}$

$= \left(\frac{1+\tan x}{1-\tan x}\right)^2$

$x \in \mathbb{R}$
$x \notin \left\{\frac{(2k+1)\pi}{4}, \frac{(2k+1)\pi}{2}\right\}$

(17) $|\sec x - \tan x| = \sqrt{\frac{1-\sin x}{1+\sin x} \cdot \frac{1+\sin x}{1-\sin x}}$

$= \sqrt{\frac{(1-\sin x)^2}{\cos^2 x}}$

$= \left|\frac{1-\sin x}{\cos x}\right|$

$= |\sec x - \tan x|$

Chapter 2

2-2

Periodic fctns $\rightarrow$ fctn of sin + cos.
Periodic fctns can be thought of as being like repeating decimals.

ex. the fraction $\frac{123}{999}$ yields .123123123....
you can see how it repeats. Every fourth is the same. a fctn repeats as well. an example of a fctn could be:

$f = \{x, f(x) : x \in J$ and $f(x)$ is the remainder after $x \div 3\}$

$f(-3) = 0$
$f(-2) = 1$
$f(-1) = 2$
$f(0) = 0$
$f(1) = 1$
$f(2) = 2$
$f(3) = 0$
$f(4) = 1$

This repeats every 3rd number. So we say it has a fundamental period of 3
This could be rewritten

$f(x+a) = f(x)$ where a is our period

ie. $f(x+3) = f(x)$ Try it.

In Trig we are interested in only a few specific periodic fctn or circular fctns.

The 1st of the fctns is called the cosine. The second is the Sine. These fctns are developed from the unit circle

The effect as described in the text is a wrappin of the x-axis around the circle. Thus if you wrapped the x-axis completely around, you would have the point $+2\pi$ on x axis lined up exactly where zero is said circumference of unit circle is 2π.

If we wrapped it around again we would have 4π at the same spot. we would get a period of 2π. That is any point would be exactly 2π from the next point that lie in the same positions on the circle.

We say the cosine has period 2π.

It can be written: $\cos(x + 2k\pi) = \cos x$.

The same idea is true for the sin fctns with the exception that we would wrap the y axis around the circle.

By definition we say $(\cos x)^2$ will be written $\cos^2 x$ or $(\sin x)^3$ will be $\sin^3 x$. Use the parenthesis only to make clear something other than the std. $\cos x$ or $\sin x$.

ex. $\cos(-x)$ or $\sin(3+2x)$ etc.

Another way to think of the Sin + Cos is in terms of u and v. of the unit circle

I equations of unit circle in x, y is
$x^2+y^2=1$ or $u^2+v^2=1$
the x in $\sin x \cos x$ etc refers to the arc length. Use (u,v) coord system thus leaving x to refer to arc length

II We use this same formula using Cos + Sin, ie. $\cos^2 x + \sin^2 x = 1$

III You will see writen $y = \cos x$, from above identity here $|\cos x| \leq 1$ look at unit circle also true for $|\sin x| \leq 1$ Thus the range on the y value in $y=\cos x$ or $y = \sin x$ is less than or equal to 1. The Domain of x is $\mathbb{R}$.

(22) Look back at ex 1-5 pg 24-25.
$f(x) = \sin x \cos x$ WTS periodic w/period 2π
If it is then $f(x+2\pi) = f(x) = \sin x \cos x$.
Thus lets evaluate $f(x)$ at $f(x+2\pi)$.
$f(x+2\pi) = \sin(x+2\pi)\cos(x+2\pi)$
but we know $\sin(x+2\pi) = \sin x$ because sin has Period 2π. Also $\cos(x+2\pi) = \cos x$ Same reason. Thus substituting equals for equals, we have $f(x+2\pi) = \sin x \cos x$
Since we know $\sin x \cos x = f(x)$ we have $f(x+2\pi) = f(x)$ which is the definition of a fctn with period 2π.

H.W. Pg 49-50 use ex as guide

1-21 all

HW Pg 49-50

① $\sin\frac{13\pi}{2} = \sin\left((2\pi)\cdot 3 + \frac{\pi}{2}\right) = \sin\frac{\pi}{2} \Rightarrow x = \frac{\pi}{2}$

② $\cos\frac{18\pi}{5} = \cos\left[(2\pi)\cdot 1 + \frac{8\pi}{5}\right) \Rightarrow x = \frac{8\pi}{5}$

③ $\sin\left(-\frac{\pi}{2}\right) = \sin\left(2\pi - \frac{\pi}{2}\right) = \sin\left(\frac{3\pi}{2}\right) \Rightarrow x = \frac{3\pi}{2}$

④ $\cos(-\pi) = \cos(2\pi - \pi) = \cos\pi \Rightarrow x = \pi$

⑤ $\cos\left(-\frac{7}{3}\pi\right) = \cos\left(2\pi - \frac{2\pi}{3}\right) = \cos\left(-\frac{\pi}{3}\right) =$
$\cos\left(2\pi - \frac{\pi}{3}\right) = \cos\left(\frac{5\pi}{3}\right) \Rightarrow x = \frac{5\pi}{3}$

⑥ $\sin\left(\frac{-13\pi}{3}\right) = \sin\left(2\pi(-3) + \frac{5\pi}{3}\right) \Rightarrow x = \frac{5\pi}{3}$

⑦ $\sin 18\pi = \sin(9\cdot 2\pi) = \sin 0 \Rightarrow x = 0$

⑧ $\cos 26\pi = \cos(13\cdot 2\pi) = \cos 0 \Rightarrow x = 0$

⑨ $\cos 13\pi = \cos(6\cdot 2\pi + \pi) = \cos\pi \Rightarrow x = \pi$

Notice ex 9 is $\frac{1}{2}$ of ex 8 but value is different! This is true of periodic fctns!

⑩ $\cos 27\pi = \cos(13\cdot 2\pi + \pi) = \cos\pi \Rightarrow x = \pi$

⑪ $\cos 4.125\pi = \cos\left(2\cdot 2\pi + \frac{\pi}{8}\right) = \cos\frac{\pi}{8} \Rightarrow x = \frac{\pi}{8}$

⑫ $\sin 2315\pi = \sin(1157\cdot 2\pi + \pi) = \sin\pi \Rightarrow x = \pi$

(13) $\cos x = \frac{\sqrt{2}}{2}$ 1st Quad

$\left(\frac{\sqrt{2}}{2}\right)^2 + \sin^2 x = 1$

$\frac{2}{4} + \sin^2 x = 1$

$\sin^2 x = \frac{1}{2}$

$\sin x = \frac{\sqrt{2}}{2}$ ← positive because

(14) $\cos x = \frac{\sqrt{2}}{2}$ 4th Quad

$\left(\frac{\sqrt{2}}{2}\right)^2 + \sin^2 x = 1$

$\frac{2}{4} + \sin^2 x = 1$

$\sin^2 x = \frac{1}{2}$

$\sin x = \frac{\sqrt{2}}{2} \Rightarrow -\frac{\sqrt{2}}{2}$

(15) $\cos x = -\frac{\sqrt{2}}{2}$ 3rd Quad

$\frac{1}{2} + \sin^2 x = 1$

$\sin^2 x = \frac{1}{2}$

$\sin x = \frac{\sqrt{2}}{2}$ ← $-\frac{\sqrt{2}}{2}$

(16) $\cos x = -\frac{\sqrt{2}}{2}$ 2nd Quad

$\sin^2 x = \frac{1}{2}$

$\sin x = \frac{\sqrt{2}}{2}$ positive.

(17) $\cos x = -\frac{\sqrt{3}}{2}$ 2nd

$\frac{3}{4} + \sin^2 x = 1$

$\sin^2 x = \frac{1}{4}$

$\sin x = \frac{1}{2}$

(18) $\cos x = -\frac{\sqrt{3}}{2}$ 3rd

$\sin^2 x = \frac{1}{4}$

$\sin x = \frac{1}{2} \Rightarrow$

$\sin x = -\frac{1}{2}$

(19) $\cos x = \frac{1}{2}$ 4th

$\frac{1}{4} + \sin^2 x = 1$

$\sin^2 x = \frac{3}{4}$

$\sin x = \frac{\sqrt{3}}{2}$

$\sin x = -\frac{\sqrt{3}}{2}$

(20) $\cos x = -\frac{1}{2}$ 2nd

$\frac{1}{4} + \sin^2 x = 1$

$\sin^2 x = \frac{3}{4}$

$\sin x = \frac{\sqrt{3}}{2}$

(21) WTS $f(x) = \sin x + \cos x$ has period 2π

If true then $f(x + 2\pi) = f(x)$ Def.

evaluate $f(x)$ at $f(x+2\pi)$

$f(x+2\pi) = \sin(x+2\pi) + \cos(x+2\pi)$

Def: $\sin(x+2\pi) = \sin x$

$\cos(x+2\pi) = \cos x$ $\therefore$

$f(x+2\pi) = \sin x + \cos x$

$\sin x + \cos x = f(x)$ $\therefore$

$f(x+2\pi) = f(x)$

2-3

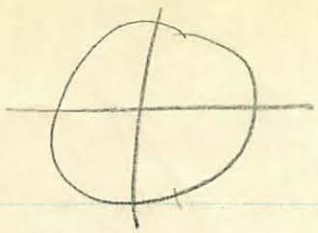
Values of $\cos x + \sin x$ for special values of x.

Rather than go into a lengthy explaination, as in the text, on locating values of the cos + sin, think again of the coordinates of a circle in terms of x, y as we did in chapt. 1.

Here we are using the x in $\cos x$ and $\sin x$ as radian measure.

Locate $x = 0 \cdot \pi = 0$ on unit circle.
What is the cos value, where are we in terms of (x, y) coordinates?
$(1, 0)$, i.e. $\cos 0 = 1$ and $\sin 0 = 0$
where is $\frac{\pi}{4}$? $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ Thus
$\cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$, $\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$.

This along with the identity:
$$\cos^2 x + \sin^2 x = 1$$
are absolute need to know information



(2)

x	sin x	cos x
0	0	1
$-\frac{\pi}{6}$	$-1/2$	$\sqrt{3}/2$
$-\frac{\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
$-\pi/3$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$-\pi/2$	-1	0
$-2\pi/3$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$
$-3\pi/4$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$
$-5\pi/6$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$
$-\pi$	0	-1
$-7\pi/6$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$
$-5\pi/4$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$
$-4\pi/3$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$
$-3\pi/2$	1	0
$-5\pi/3$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$-7\pi/4$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
$-11\pi/6$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
-2π	0	1

(3) $\cos 3\pi = -1$

(4) $\sin 5\pi = 0$

(5) $\sin \frac{9\pi}{4} = \frac{\sqrt{2}}{2}$

(6) $\cos \frac{10\pi}{3} = \frac{1}{2}$

(7) $\cos \frac{83\pi}{6} = \frac{\sqrt{3}}{2}$

(8) $\sin \frac{1088\pi}{8} = -1$

(9) $\sin -11\pi = 0$

(10) $\cos \frac{-13\pi}{2} = 0$

(11) $\cos \frac{-15\pi}{2} = 0$

(12) $\sin\left(-\frac{27\pi}{6}\right) = 0$ $\frac{9}{2}$

(13) $\sin \frac{-21\pi}{4} = \frac{\sqrt{2}}{2}$

(14) $\cos\left(\frac{-82\pi}{3}\right) = -\frac{1}{2}$

12
6|83
23
12
11

12
8|108
5
28 3/2
16
12

26
3|82
22
18
4
3

Ex 2-4 Pg 61-63

① $\cos\frac{5\pi}{12} = \cos\left(\frac{\pi}{6}+\frac{\pi}{4}\right) =$

$\cos\frac{\pi}{6}\cos\frac{\pi}{4} - \sin\frac{\pi}{6}\sin\frac{\pi}{4}$

$\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} - \frac{1}{2}\cdot\frac{\sqrt{2}}{2} =$

$\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6}-\sqrt{2}}{4}$

② $\cos\frac{11\pi}{12} = \cos\left(\frac{2\pi}{3}+\frac{\pi}{4}\right) =$

$\cos\frac{2\pi}{3}\cos\frac{\pi}{4} - \sin\frac{2\pi}{3}\sin\frac{\pi}{4}$

$-\frac{1}{2}\cdot\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$

$-\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \frac{-\sqrt{2}-\sqrt{6}}{4}$

③ $\cos\frac{13\pi}{12} = \cos\left(\frac{3\pi}{4}+\frac{\pi}{3}\right) =$

$\cos\frac{3\pi}{4} + \cos\frac{\pi}{3} - \sin\frac{3\pi}{4}\sin\frac{\pi}{3}$

$-\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2}$

$-\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = -\frac{\sqrt{6}-\sqrt{2}}{4}$

④ $\cos\frac{17\pi}{12} = \cos\left(\frac{3\pi}{4}+\frac{2\pi}{3}\right) =$

$\cos\frac{3\pi}{4}\cos\frac{2\pi}{3} - \sin\frac{3\pi}{4}\sin\frac{2\pi}{3}$

$-\frac{\sqrt{2}}{2}\cdot-\frac{1}{2} - \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}$

$\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \frac{\sqrt{2}-\sqrt{6}}{4}$

⑤ $\cos\left(-\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{3}-\frac{\pi}{4}\right) =$

$\cos\frac{\pi}{3}\cos\frac{\pi}{4} \oplus \sin\frac{\pi}{3}\sin\frac{\pi}{4} =$

$\frac{1}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} =$

$\frac{\sqrt{2}+\sqrt{6}}{4}$

⑥ $\cos\left(-\frac{5\pi}{12}\right) = \cos\left(\frac{\pi}{3}-\frac{3\pi}{4}\right) =$

$\cos\frac{\pi}{3}\cos\frac{3\pi}{4} + \sin\frac{\pi}{3}\sin\frac{3\pi}{4} =$

$\frac{1}{2}\cdot-\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} =$

$\frac{\sqrt{6}-\sqrt{2}}{4}$

Pg 61

(7) $\cos\left(\frac{-13\pi}{12}\right) = \cos\frac{13\pi}{12} = \cos\left(\frac{3\pi}{4} + \frac{\pi}{3}\right)$ NOW SAME AS #3

(8) $\cos\left(-\frac{17\pi}{12}\right) = \cos\frac{17\pi}{12} = \cos\left(\frac{3\pi}{4} + \frac{2\pi}{3}\right)$ NOW SAME AS #4

(9) $\cos\pi\cos\frac{2\pi}{3} + \sin\pi\sin\frac{2\pi}{3} =$

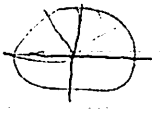
$\cos\left(\pi - \frac{2\pi}{3}\right) = \cos\frac{\pi}{3} = \frac{1}{2}$

(10) $\cos\frac{2\pi}{3}\cos\frac{\pi}{6} - \sin\frac{2\pi}{3}\sin\frac{\pi}{6} =$

$\cos\left(\frac{2\pi}{3} + \frac{\pi}{6}\right) = \cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$

(11) $\cos\pi\cos\frac{\pi}{4} + \sin\pi\sin\frac{\pi}{4} =$

$\cos\left(\pi - \frac{\pi}{4}\right) = \cos\frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$



(12) $\cos\frac{3\pi}{4}\cos\frac{\pi}{4} + \sin\frac{3\pi}{4}\sin\frac{\pi}{4} =$

$\cos\left(\frac{3\pi}{4} - \frac{\pi}{4}\right) = \cos\frac{\pi}{2} = 0$

(13) $\cos\pi\cos\left(-\frac{\pi}{3}\right) + \sin\pi\sin\left(-\frac{\pi}{3}\right) =$

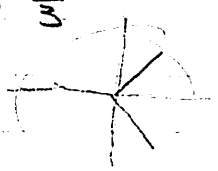
$\cos\left(\pi + \frac{\pi}{3}\right) = \cos\frac{4\pi}{3} = -\frac{1}{2}$

Notice $\cos -\frac{\pi}{3} = \cos\frac{\pi}{3}$ and $\sin\left(-\frac{\pi}{3}\right) = -\sin\frac{\pi}{3}$

thus original pblm becomes $\cos\pi\cos\frac{\pi}{3} - \sin\pi\sin\frac{\pi}{3}$

(22) $\cos\frac{\pi}{6}\cos\left(-\frac{\pi}{3}\right) - \sin\frac{\pi}{6}\sin\left(-\frac{\pi}{3}\right)$

$\cos\left(\frac{\pi}{6} - \frac{\pi}{3}\right) = \cos\left(-\frac{\pi}{3}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$



(23) $\cos(-\pi)\cos\left(-\frac{\pi}{4}\right) - \sin(-\pi)\sin\left(-\frac{\pi}{4}\right) =$

$\cos\pi\cos\frac{\pi}{2} - \sin\pi\sin\frac{\pi}{4} =$

$-1 \cdot \frac{\sqrt{2}}{2} = -\frac{\sqrt{2}}{2}$

(24) $\cos\left(-\frac{\pi}{3}\right)\cos\left(-\frac{\pi}{3}\right) + \sin\left(-\frac{\pi}{3}\right)\sin\left(-\frac{\pi}{3}\right) =$

$\cos\frac{\pi}{3}\cos\frac{\pi}{3} + \sin\frac{\pi}{3}\sin\frac{\pi}{3} =$

$\cos\left(\frac{\pi}{3} - \frac{\pi}{3}\right) = \cos 0 = 1$

(25) $\cos\left(x + \frac{\pi}{3}\right)\cos\frac{\pi}{6} - \sin\left(x + \frac{\pi}{3}\right)\sin\frac{\pi}{6} =$

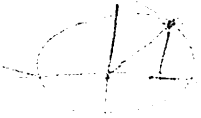
$\cos\left(x + \frac{\pi}{3} + \frac{\pi}{6}\right) = \cos\left(x + \frac{\pi}{2}\right) =$

$\cos x \cos\frac{\pi}{2} - \sin x \sin\frac{\pi}{2}$

$-\sin x \cdot 1 = -\sin x$

(26) $\cos\left(x + \frac{\pi}{3}\right)\cos x + \sin\left(x + \frac{\pi}{3}\right)\sin x =$

$\cos\left(x + \frac{\pi}{3} - x\right) = \cos\frac{\pi}{3} = \frac{1}{2}$



HW Pg 66

① $\cos\left(\frac{\pi}{3}+x\right) = \cos\frac{\pi}{3}\cos x - \sin\frac{\pi}{3}\sin x =$
$\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x$

② $\cos\left(\frac{\pi}{6}-x\right) = \cos\frac{\pi}{6}\cos x + \sin\frac{\pi}{6}\sin x$
$\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin$

③ $\cos\left(\frac{3\pi}{2}-x\right) = \cos\frac{3\pi}{2}\cos x + \sin\frac{3\pi}{2}\sin x = -\sin x$

④ $\cos\left(\frac{3\pi}{2}+x\right) = \cos\frac{3\pi}{2}\cos x - \sin\frac{3\pi}{2}\sin x = \sin x$

⑤ $\cos(x-\pi) = \cos x\cos\pi + \sin x \sin\pi = -\cos x$

⑥ $\cos(2\pi - x) = \cos(-x) = \cos x$

⑦ $\cos\left(x-\frac{\pi}{2}\right) = \cos x \cos\frac{\pi}{2} + \sin x \sin\frac{\pi}{2} = \sin x$

⑧ $\cos\left(x-\frac{3\pi}{2}\right) = \cos x\cos\frac{3\pi}{2} + \sin x \sin\frac{3\pi}{2} = -\sin x$

⑨ $\cos(\pi+x) = \cos\pi\cos x - \sin\pi\sin x = -\cos x$

⑩ $\cos(\pi - x) = \cos\pi\cos x + \sin\pi\sin x = -\cos x$

⑪ add $\cos^2 x$

⑫ add $2\sin^2 x$

⑬ add 1, add $\sin^2 x$, sub $\cos^2 x$

⑭ add $2\sin^2 x$, add 1 divide by 2

⑮ $\cos\left(\frac{\pi}{2}-x\right) = \sin x$ substitute + multiply.
$\sin\left(\frac{\pi}{2}-x\right) = \cos x$

⑯ $\cos(-x) = \cos x$ ditto ↑
$\sin(-x) = -\sin x$

⑰ $\cos(x_2 - x_1) - \cos(x_2 + x_1) = 2\sin x_2 \sin x_1$,
$\cos x_2\cos x_1 + \sin x_2 \sin x_1 =$
$-\cos x_2\cos x_1 + \sin x_2 \sin x_1 =$
$2\sin x_2 \sin x_1 = 2\sin x_2 \sin x_1$

⑱ $\cos(x-\pi) - \cos(x+\pi) = 0$
$\cos x\cos\pi + \sin x\sin\pi - \cos x\cos\pi + \sin x\sin\pi = 0$

19 $\cos^2 x_2 \cos^2 x_1 - \sin^2 x_2 \sin^2 x_1 = \cos^2 x_2 - \sin^2 x_1$

$\cos^2 x_2 (1 - \sin^2 x_1) - (1 - \cos^2 x_2)$ $\qquad$ $\cos(x_2 + x_1)$

$\cos^2 x_2 - \cos^2 x_2 \sin^2 x_1 - \sin^2 x_1 + \sin^2 x_1 \cos^2 x_2 =$

$\cos^2 x_2 - \sin^2 x_1 =$

do as below.

(19) $\cos(x_2 - x_1)\cos(x_2 + x_1) = \cos^2 x_2 - \sin^2 x_1$

cos +, −

$[\cos x_2 \cos x_1 + \sin x_2 \sin x_1][\cos x_2 \cos x_1 - \sin x_2 \sin x_1] =$

$\cos^2 x_2 \cos^2 x_1 - \sin^2 x_2 \sin^2 x_1 =$

→ $\cos^2 x_2 \cos^2 x_1 + \cos^2 x_2 \sin^2 x_1 - \cos^2 x_2 \sin^2 x_1 - \sin^2 x_2 \sin^2 x_1$

$\cos^2 x_2 (\cos^2 x_1 + \sin^2 x_1) - \sin^2 x_2 (\cos^2 x_2 + \sin^2 x_2)$

$\cos^2 x_2 - \sin^2 x_2$.

TRICKY PROBLEM You must add & subtract the same quantity in such a way as to match up the $\cos^2 x$'s and $\sin^2 x$'s.

20 $\cos^3 x + \sin^3 x = \cos x(1 - \cos x \sin x) + \sin x(1 - \cos x \sin x)$

use hint

$(\cos x + \sin x)(\underbrace{\cos^2 x - \cos x \sin x + \sin^2 x}_{=1})$

& distribute

(21) $\cos(\frac{\pi}{2} + x_1 - x_2) = \cos x_1 \sin x_2 - \sin x_1 \cos x_2$

regroup as:

$\cos[(\frac{\pi}{2} - x_2) + x_1] =$

$\cos(\frac{\pi}{2} - x_2)\cos x_1 - \sin(\frac{\pi}{2} - x_2)\sin x_1$

~~use~~ identities pg 61 $\cos(\frac{\pi}{2} - x_2) = \sin x_2$; $\sin(\frac{\pi}{2} - x_2) = \cos x_2$

$\sin x_2 \cos x_1 - \cos x_2 \sin x_1$

(22) regroup (21): $\cos(\frac{\pi}{2} - (x_2 - x_1)) = \cos\frac{\pi}{2}\cos(\ ) + \sin\frac{\pi}{2}\sin(x_2 - x_1) =$

$\sin x_2 - x_1$

But since $\cos(\frac{\pi}{2} - (x_2 - x_1)) = \cos(\frac{\pi}{2} + x_1 - x_2) = \cos x_1 \sin x_2 - \sin x_1 \cos x_2$ Then:

$\sin(x_2 - x_1) = \cos x_1 \sin x_2 - \sin x_1 \cos x_2$

(19)

Soln. $\cos^2 x_2 \cos^2 x_1 - \sin^2 x_2 \sin^2 x_1 = \cos^2 x_2 - \sin^2 x_1$

$\cos^2 x_2 \cos^2 x_1 - \cos^2 x_2 = \sin^2 x_2 \sin^2 x_1 - \sin^2 x_1$

$\cos^2 x_2 (\cos^2 x_1 - 1) = \sin^2 x_1 (\sin^2 x_2 - 1)$

$\cos^2 x_2 (1 - \sin^2 x_1 - 1) = \sin^2 x_1 (1 - \cos^2 x_2 - 1)$

$-\cos^2 x_2 \sin^2 x_1 = -\sin^2 x_1 \cos^2 x_2$

Identity.

2-6

Addition properties of sin.

Pblm #22 pg 66 illustrates how the sin addition formula is generated.

You should make an effort to learn the numbered equations on Page 66-67

Read ex Pg 69.

HW Pg 69-71

1, 2, 4, 9-15, 41

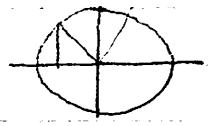
Pg 69

① $\sin\frac{\pi}{12} = \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \sin\frac{\pi}{3}\cos\frac{\pi}{4} - \sin\frac{\pi}{4}\cos\frac{\pi}{3}$

$\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}$

② $\sin\frac{5\pi}{12} = \sin\left(\frac{2\pi}{3} - \frac{\pi}{4}\right) = \sin\frac{2\pi}{3}\cos\frac{\pi}{4} - \sin\frac{\pi}{4}\cos\frac{2\pi}{3} =$

$\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} - \left(\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right) = \frac{\sqrt{6}+\sqrt{2}}{4}$



④ $\sin\left(-\frac{5\pi}{12}\right) = -\sin\left(\frac{5\pi}{12}\right) = \frac{-\sqrt{6}-\sqrt{2}}{4}$

⑨ $\cos\frac{2\pi}{5} = \cos\left(\pi + \frac{2\pi}{5}\right) = -\cos\frac{2\pi}{5}$

⑩ $\sin\frac{7\pi}{5} = \sin\left(\pi + \frac{2\pi}{5}\right) = -\sin\frac{2\pi}{5}$

⑪ $\sin\frac{8\pi}{7} = \sin\left(\pi + \frac{\pi}{7}\right) = -\sin\frac{\pi}{7}$

⑫ $\cos\frac{8\pi}{7} = \cos\left(\pi + \frac{\pi}{7}\right) = -\cos\frac{\pi}{7}$

⑬ $\sin\frac{11\pi}{7} = \sin\left(\pi - \frac{4\pi}{7}\right) = \sin\left(-\frac{4\pi}{7}\right) = -\sin\frac{4\pi}{7}$

⑭ $\cos\frac{11\pi}{7} = \cos\left(\pi - \frac{4\pi}{7}\right) = \cos\left(-\frac{4\pi}{7}\right) = \cos\frac{4\pi}{7}$

⑮ $\cos\left(-\frac{14\pi}{11}\right) = \cos\left(\frac{14\pi}{11}\right) = \cos\left(\pi + \frac{3\pi}{11}\right) = -\cos\frac{3\pi}{11}$

	$-x$	$\frac{\pi}{2}+x$	$\frac{\pi}{2}-x$	$x-\frac{\pi}{2}$	$\pi+x$	$\pi-x$	$\frac{3\pi}{2}+x$	$\frac{3\pi}{2}-x$	$x-\frac{3\pi}{2}$
sin	$-\sin x$	$\cos x$	$\cos x$	$-\cos x$	$-\sin x$	$\sin x$	$-\cos x$	$-\cos x$	$\cos x$
cos	$\cos x$	$-\sin x$	$\sin x$	$\sin x$	$-\cos x$	$-\cos x$	$\sin x$	$-\sin x$	$-\sin x$

$\sin(x-\frac{\pi}{2}) = \sin x \cancel{\cos\frac{\pi}{2}}^{0} - \sin\frac{\pi}{2}\cos x = -\cos x$

$\sin(\pi+x) = \cancel{\sin\pi}^{0}\cos x + \sin x \cos\pi = -\sin x$

$\sin(\pi-x) = -\sin x\cos\pi = \sin x$

$\sin(\frac{3\pi}{2}+x) = \sin\frac{3\pi}{2}\cos x + \sin x \cancel{\cos\frac{3\pi}{2}}^{0} = -\cos x$

$\sin(\frac{3\pi}{2}-x) = \sin\frac{3\pi}{2}\cos x - \sin x \cancel{\cos\frac{3\pi}{2}}^{0} = -\cos x$

$\sin(x-\frac{3\pi}{2}) = \sin x \cancel{\cos\frac{3\pi}{2}}^{0} - \sin\frac{3\pi}{2}\cos x = \cos x$

$\cos(x-\frac{\pi}{2}) = \cos x \cancel{\cos\frac{\pi}{2}}^{0} + \sin x \sin\frac{\pi}{2} = \sin x$

$\cos(\pi+x) = \cos x\cos\pi - \sin x \cancel{\sin\pi}^{0} = -\cos x$

$\cos(\pi-x) = \cos\pi\cos x + \cancel{\sin\pi}^{0}\sin x = -\cos x$

$\cos(\frac{3\pi}{2}+x) = \cos\frac{3\pi}{2}\cos x - \sin\frac{3\pi}{2}\sin x = \sin x$

$\cos(\frac{3\pi}{2}-x) = \cos(x-\frac{3\pi}{2}) = \cos x \cancel{\cos\frac{3\pi}{2}}^{0} + \sin x \sin\frac{3\pi}{2} = -\sin x$

Which of the following are Identities?

① $\tan^2 x - \sin^2 x = (\tan^2 x)(\sin^2 x)$

② $\dfrac{\cos 4x}{1+\sin 4x} = \sec 4x - \tan 4x$

③ $\frac{1}{6}(\sin 4x)(\cos 2x) - \frac{1}{3}\cos 4x \sin 2x = \frac{1}{3}\sin^3 2x$

④ $\sin x = \dfrac{2\tan \frac{x}{2}}{1+\tan^2 \frac{x}{2}}$

change #3 to $\frac{1}{3}\sin^2 2x$ or $\frac{1}{3}\sin^3 3x$ or something so students can see what should happen when faced with an identity which is not true. So Left side will end up equal

(1) $\text{TAN}^2x - \sin^2x = (\tan^2x)(\sin^2x)$

$\frac{\sin^2x}{\cos^2x} - \sin^2x =$

$\frac{\sin^2x(1-\cos^2x)}{\cos^2x} =$

$\frac{\sin^2x \cdot \sin^2x}{\cos^2x} =$

$\frac{\sin^2x}{\cos^2x} \cdot \sin^2x =$

$\tan^2x \sin^2x =$

Restrictions:

$\cos^2x \neq 0$

$x \notin \left\{\frac{(2k+1)\pi}{2}\right\}$

(2) $\frac{\cos 4x}{1+\sin 4x} = \sec 4x - \tan 4x$

$\frac{1-\sin 4x}{1-\sin 4x}(\quad) =$

$\frac{\cos 4x - \cos 4x \sin^4 4x}{1-\sin^2 4x} =$

$\frac{\cos 4x(1-\sin 4x)}{\cos^2 4x}$

$\frac{1}{\cos^2 4x} - \frac{\sin^4 x}{\cos^2 4x}$

$\sec 4x - \tan 4x =$

$4x \notin \left\{\frac{(2k+1)\pi}{2}\right\}$

$x \notin \left\{\frac{(2k+1)\pi}{8}\right\}$

$\sin 4x = -1$

when $4x = \frac{3\pi}{2}$

$x = \frac{3\pi}{8} + 2k\pi$

$\frac{3\pi + 16k\pi}{8}$

$x \notin \left\{\frac{(3+16k)\pi}{8}, \frac{\pi+2k\pi}{8}\right\}$

$\cos 4x \neq 0$

$4x = \frac{\pi}{2} + k\pi$

$x = \frac{\pi}{8} + \frac{k\pi}{4}$

#3 $\frac{1}{6}\sin 4x\cos 2x - \frac{1}{3}\cos 4x\sin 2x = \frac{1}{3}\sin^3 2x$

$\frac{1}{6}(\sin(2x+2x)\cos 2x) - \frac{1}{3}(\cos(2x+2x)\sin 2x) =$

$\frac{1}{6}(2\sin 2x\cos^2 2x) - \frac{1}{3}\cos^2 2x\sin 2x + \frac{1}{3}\sin^3 2x =$

$\frac{1}{3}\sin 2x\cos^2 2x - \frac{1}{3}\cos^2 2x\sin 2x + \frac{1}{3}\sin^3 2x =$

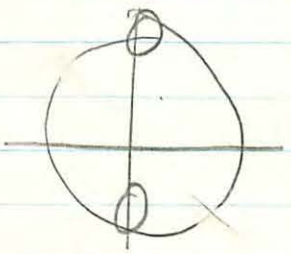
$\frac{1}{3}\sin^3 2x = \frac{1}{3}\sin^3 2x$

#4 $\sin x = \dfrac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$= \dfrac{\frac{2\sin\frac{x}{2}}{\cos\frac{x}{2}}}{1+\frac{\sin^2\frac{x}{2}}{\cos^2\frac{x}{2}}} = \dfrac{\frac{2\sin\frac{x}{2}}{\cos\frac{x}{2}}}{\frac{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}}{\cos^2\frac{x}{2}}} = 2\sin\frac{x}{2}\cos\frac{x}{2} =$

$= \sin\left(\frac{x}{2}+\frac{x}{2}\right)$

$= \sin x.$



$\frac{x}{2} \neq \frac{(2k+1)\pi}{2}$

$x \neq (2k+1)\pi$

10 min

5 = 4
4 = V
3 = 0
2 = -
0,1 = =

Late Quiz

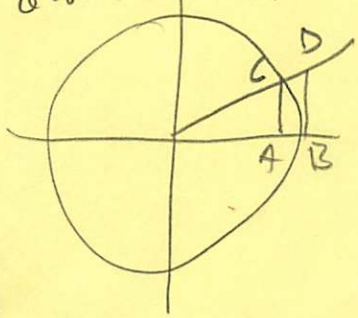
I Given $\cos A = \frac{-3}{5}$; $\pi < A < \frac{3\pi}{2}$

$\cos B = \frac{5}{13}$; $\frac{3\pi}{2} < B < 2\pi$

A) what Quad is $A - B$?

B) what Quad is $A + B$?

II Given unit circle



Determine coord of A, B, C, D

III Prove

A) $1 - \tan^4 x = 2\sec^2 x - \sec^4 x$

B) $\csc x + \cot x = \dfrac{1}{\csc x - \cot x}$

I

$\cos A = -\frac{3}{5}$ 3rd Quad $\quad \cos B = \frac{5}{13}$ 4th Quad

$\sin A = -\frac{4}{5}$ $\quad \sin B = -\frac{12}{13}$

$(A+B)$ cos/sin + ∴ 1st Quad

$(A-B)$ cos/sin − ∴ 3rd Quad.

$$\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A$$

$$-\frac{4}{5}\ \frac{5}{13} \pm \frac{-12}{13}\ \frac{-3}{5}$$

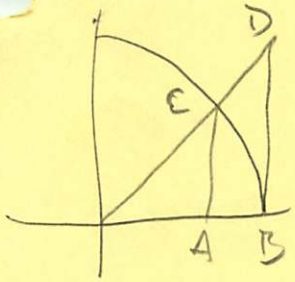
$$\frac{-20 \pm 36}{65} \qquad + \frac{16}{65} \qquad - \frac{-56}{65}$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$-\frac{3}{5}\ \frac{5}{13} \mp \frac{-4}{5}\ \frac{-12}{13}$$

$$\frac{-15 \mp 48}{65} \qquad + \frac{33}{65} \qquad - \frac{-63}{65}$$

II



A) $(\cos x, 0)$

B) $(1, 0)$

C) $(\cos x, \sin x)$

D) $(1, \tan x)$

all required for 1 right.

III

A) $1 - \tan^4 x = 2\sec^2 x - \sec^4 x$

$s^2 + c^2 = 1$ $\quad (1 - \tan^2 x)(1 + \tan^2 x) =$

$t^2 + 1 = \sec^2$ $\quad (1 - (\sec^2 x - 1))\sec^2 x =$

$t^2 = \sec^2 - 1$ $\quad (1 - \sec^2 x + 1)$

$(2 - \sec^2 x)\sec^2 x =$

B) $\csc x - \cot x = \frac{1}{\csc x - \cot x}$

$= \frac{(\csc x + \cot x)}{\csc x^2 - \cot x^2}$

$s^2 + c^2 = 1$

$1 + \cot^2 = \csc^2$

$1 = \csc^2 x - \cot^2 x$

$= \csc x + \cot x$

$\frac{(\csc x + \cot x)}{(\csc x + \cot x)}$